

The Assignment and Division of the Tax Base in a System of Hierarchical Governments

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Abstract

The existence of either “horizontal” fiscal externalities, in which changes in one jurisdiction’s policies affect the government budget of other jurisdictions and therefore the utility of its residents or “vertical” externalities, in which changes in one level of government’s policies affect the budget of another level of government, may lead to non-optimal government policies. These fiscal externalities, then, suggest the possibility of corrective policies.

The focus here is on vertical externalities. In a growing literature, these externalities are associated with the extent that tax bases are shared or “co-occupied” by two different levels of government. Given that co-occupancy is the cause of or at least exacerbates the externality, I consider, the optimal “assignment” of the tax base and, more specifically, whether the co-occupancy of tax bases is desirable. Specifically, I examine the optimal extent of the tax base of a lower level of government (local) and a higher level (state) in a hierarchical system of governments. The co-occupancy of the tax base influences the magnitude and possible the direction of "vertical" fiscal externalities associated with the taxes of one or both of the levels of government. Using a model in which there is a continuum of commodities, each with the same demand characteristics, I formally consider whether, as has been asserted in a number of studies, whether it is optimal to eliminate all co-occupancy between the tax bases of the two levels of government.

While I find that it is indeed not optimal to have co-occupancy in the tax base in the absence of other corrective policies for the fiscal externality, eliminating co-occupancy does not, in general, eliminate fiscal externalities, meaning that tax rates can still be above or now below the socially-optimal level. Thus elimination of co-occupancy in the tax base is not a substitute for a policy such as intergovernmental matching grants which directly eliminates fiscal externalities. If alternative policies are available such as matching grants that do eliminate fiscal externalities and governments are restricted to set the same tax rate on all commodities in their base, the optimal division of the tax base changes dramatically – optimality requires both governments tax the entire base. (JEL H77 - Intergovernmental Relations; Federalism; Keywords: federalism, fiscal competition, assignment, vertical externalities)

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1. Introduction

The concept of a "horizontal" fiscal externality arising from "tax competition" among governments at the same level has been the topic of numerous papers in the past twenty-five years.¹ This literature focuses on the impact that one jurisdiction's taxes has on the welfare of residents in other jurisdictions. Increases in taxes on a mobile tax base in one jurisdiction will lead to a decrease in the tax base (usually mobile capital) there but increases in the other jurisdictions' tax bases. This "horizontal" fiscal externality is ignored by the jurisdiction raising (or lowering) its taxes. Because it is a positive externality, the jurisdictional governments, if choosing policies to maximize the utility of their residents, will tax capital at too low a rate and underprovide public services. A number of studies, including Arnott and Grieson (1981), Gordon (1983), Wildasin (1984, 1989), Dahlby and Wilson (1994), Hoyt (1991), and Hoyt and Jensen (1996), have considered policies, most frequently intergovernmental grants, that might be employed by a higher level of government to "correct" horizontal fiscal externalities.

More recently another fiscal externality, a "vertical" fiscal externality, has come to attention of researchers beginning with Johnson (1988) and Flowers (1988) and continuing with Dahlby (1994,1996), Keen and Kotsogiannis (1996), Boadway and Keen (1996), Boadway, et. al. (1998), Keen (1998), Hoyt (2001), Dahlby and Wilson (2003), Wrede (1996, 2000), and Wilson and Janeba (2005) among others. As the name "vertical" implies, this externalities arises between governments at different levels, for example, between state or provincial governments and local governments or federal and state or provincial governments. In this case the focus is on the "overlap" in the tax bases of two levels of government. An example from Dahlby (1996) empirically examined by Besley and Rosen (1998) is the excise taxes placed on cigarettes by both the federal and state governments in the United States. Each state, when choosing its tax rate, presumably only considers the revenue it collects from the tax and the costs of the tax to its residents. Adopting the terminology of Dahlby (1996), the state will equate the benefits from this revenue to the private marginal cost of public funds (*MCF*) from the excise tax. However, an increase in the state excise tax not only has an impact on the state tax revenues, but possibly other states' tax revenues due to cross-border shopping (a horizontal externality) and federal tax revenues by

¹See Wilson (1999) for an extensive review of the literature on tax competition.

reducing the demand for cigarettes and therefore the tax base for the federal government. This means the social marginal cost of funds (*SMCF*) differs from the *MCF* because of these externalities. While the horizontal fiscal externality is positive, the vertical externality is negative as it reduces federal revenues. The vertical externality, then, will lead to the state overtaxing cigarettes.

As with the horizontal fiscal externalities, a number of studies (Flowers (1988), Dahlby (1996), Boadway and Keen (1996), Keen and Kotsogiannis (1996), Boadway et. al. (1998), and Hoyt (2001) among others) have considered policies by the higher level of government to correct for the vertical externalities imposed by taxation of the lower level of government. Corrective policies include separating the tax bases of the two levels of government (Flowers, 1988); increasing the number of lower-level governments (Keen, 1995; Keen and Kotsogiannis, 1996); and providing intergovernmental grants (Dahlby, 1996; Boadway and Keen, 1996; Boadway, et. al., 1998; and Flochel and Madies, 2002).

With the exception of Dahlby (1996), Hoyt (2001), some of the discussion in Keen (1998) and Dahlby et. al. (2000) which examines both a profits and labor income tax, vertical fiscal externalities have been examined in the context of a single tax base, generally labor income, shared or "co-occupied" by two different levels of government. Of course, this is a simplification as in most countries governments rely on a number of different tax bases and instruments. While this paper also examines the tax policies in a hierarchical system of government in which vertical externalities exist, it departs from previous studies in a several ways. First, rather than considering the implications of vertical externalities on tax policies when there is a single tax base that serves as the source of revenue for both levels of government (state and local), tax policy is considered with multiple tax bases. Specifically, I consider a large number (a continuum) of commodities to include in either or both of the two levels of governments' tax bases. The consideration of multiple commodities enables me to address the question of central interest to this paper -- how should the tax base be allocated between the two levels of government?

Vertical fiscal externalities act in both directions -- state taxes affect local revenues and local taxes affect state revenues. A number of studies (Flowers (1988), Keen (1995), Keen and Kotsogiannis (1996), Wrede (1996) for example) assume that both levels of government, when setting their tax policies, ignore the vertical external-

ity imposed on the other level of government. This, then, will lead to excessive taxation at both levels of government if the governments provide a public service consumed by immobile residents.² In Hoyt (2001) I also considered the case in which the higher level of government, the state, considers the impacts of its tax policies on the revenues of the local government, social-welfare maximizing policies. Here I consider both the possibility that the state government fully considers the impact of its tax policies on local revenues and the possibility that it does not fully account for or, at the extreme, ignores the impact its policies have on local tax revenues. That a higher level of government considers the impact of its grant or transfer policies, policies designed to correct externalities, on the policies of a lower level of government has been the focus of numerous studies including Arnott and Grieson (1981), Gordon (1983), Dahlby and Wilson (1994), Dahlby (1996), Boadway and Keen (1996), Wildasin (1984, 1989), and Boadway et. al. (1998). Hoyt (2001) demonstrates how the state government can use its tax policy, in the absence of grants, to ameliorate the impacts of vertical fiscal externalities on social welfare. While Hoyt (2001) focuses on the structure of taxes given the existing tax bases of the two levels of government, here I consider how the tax base should be designed to limit these fiscal externalities and maximize social welfare.

In addition to having far more than a single tax base and uniform taxation of that tax base, different levels of governments rely on very different sources of revenue. *Table 1* gives a breakdown of the different sources of revenue for U.S. state, local, and the federal government aggregated to the national level for fiscal year 2000. For state and local governments, there is only limited overlap or "co-occupancy" in sources of revenue, though, undoubtedly, there is a strong link among the alternative tax bases. For example, changes in the personal income tax, primarily a source of revenue for state governments, will undoubtedly affect property tax revenue, primarily a local source of revenue. In contrast, there is much more apparent co-occupancy of the federal and state tax bases. In contrast to the rather limited empirical work (at least relative to the voluminous theoretical work) horizontal fiscal externalities, a number of studies have examined how changes in tax rates for one level of government affect tax rates for another level of government.³ Besley and Rosen (1998) find that in-

²Overprovision need not be the result if governments provide public inputs in production as in Wrede (2000) and Dahlby and Wilson (2003).

³For a summary of the empirical work on horizontal fiscal competition see Brueckner (2005).

creases in federal tax rates on gasoline in the US have significant positive impacts on state gasoline tax rates. Similarly Esteller-More and Sole-Olle (2001) find that increases in effective US federal tax rates increase US state income and sales tax rates; in contrast to the findings of Esteller-More and Sole Olle (2001), Goodspeed (2000), in a study using a panel of national and lower local income tax for 13 OECD countries, finds that higher national income tax rates lead to lower local income tax rates. This inverse relationship between the income tax rates of higher and lower level governments is also found in a panel study of the Swedish local and regional public sectors by Andersson et. al. (2004).

The issue addressed here, what level of government should tax what goods or services or inputs is referred to in the federalism literature as the “assignment” problem. In a surprisingly small literature, the best known discussion of the appropriate assignment of the tax base in a system of hierarchical governments is found in Musgrave (1983) with nice summaries found in Musgrave and Musgrave (1989), Oates (1994), and Keen (1998). Musgrave’s “principles” for tax assignment are that: 1) “highly” progressive taxes, particularly for the use of redistribution, should be done at a higher level of government; 2) lower-level governments should avoid taxes on highly mobile tax bases focusing on less mobile sources such as land; 3) the higher level government should be responsible for taxing inequitably distributed resources; and 4) “benefit” taxes and user fees should be especially prevalent for lower-level governments.⁴ Musgrave did not address the issue of vertical fiscal externalities and how they might affect assignment. Keen (1998) does devote some discussion (and analysis) to co-occupancy and assignment by addressing the question of whether it is better to co-occupy an inelastic tax base, such as gasoline, or a more elastic tax base? Contrary to what might be the expected answer, that it is preferred to occupy an inelastic tax base in which changes in tax rates have less of an effect on the shared base, Keen shows that, in fact, the preferred base for co-occupancy is the more elastic base.

Here I address the assignment question using a very different framework from that of either Musgrave (1983) or Keen (1998). Rather than considering the type of tax base that should be taxed by different levels of government, I consider how to divide a uniform tax base among two levels of government and whether co-occupancy is desirable or not. This framework, I believe, helps focus on the question of whether the existence

⁴ This summary borrows heavily from those of Musgrave and Musgrave (1989) and Oates (1994).

of vertical fiscal externalities and the associated overprovision of public services might, as suggested by, among others, Flowers (1988), lead to the conclusion that there should be no or very limited co-occupancy among the tax bases of different levels of government. I confirm Flower's conjecture that if optimally chosen, that is if the tax base is allocated among the two levels of government in a way to maximize social welfare, no co-occupancy is indeed socially optimal in the absence of any instruments such as matching grants that eliminate any fiscal externalities. However, eliminating co-occupancy will not, in general, eliminate vertical fiscal externalities. Then even if co-occupancy is eliminated, if cross-price elasticities are nonzero, the tax rates set by the two levels of government will not be optimal. If the commodities in the tax base are gross substitutes, the elimination of co-occupancy, while not eliminating the fiscal externality, will change it from being negative (if co-occupancy was extensive) to becoming positive, meaning that tax rates would change from being "too" high to being "too" low. Alternatively, with gross substitutes, if the overlap in tax bases is set to ensure the fiscal externality associated with tax rates is eliminated, the division of the tax base is still not optimal -- social welfare is increased by eliminating the co-occupancy. While the division of the tax base between the two levels of governments obviously influences the vertical externalities associated with the tax rates, the extent and direction of the fiscal externalities associated with tax increases and the fiscal externalities associated with increases in tax bases can be quite different. In fact, it is possible, even likely, that with limited overlap there are positive fiscal externalities associated with tax increases but with any overlap in tax bases, regardless of the relationship between commodities, any increase in the overlap of the two tax bases decreases social welfare. While the elimination of co-occupancy is social-welfare improving, it is because of the fiscal externalities associated with tax bases and not tax rates. Eliminating inefficiencies arising from fiscal externalities associated with tax rates must be addressed through other corrective policies.

While I show that it is not desirable to allocate the tax base among the two governments to eliminate the fiscal externalities from taxes, elimination of these fiscal externalities will dramatically change the optimal structure of the two governments' tax bases. Here I show that if matching grants are used to eliminate the vertical fiscal externalities associated with tax increases, then given the requirement that all commodities be taxed at the same rate by each level government, social welfare is maximized only when both governments tax all com-

modities – the entire tax base is co-occupied. Thus the policy of eliminating co-occupancy is only desirable if the fiscal externalities associated with tax rates cannot be eliminated using other instruments.

In *Section 2* I outline the model and framework for analysis. In *Section 3* I consider the tax rates and tax bases that would be chosen by both levels of government if these government could, in fact, choose the base they tax. The optimal tax base for the different levels of government is considered in *Section 4*. In this section I first consider the question of how to divide the tax base between the two levels of government in the absence of any overlap. I then consider whether and under what conditions, would co-occupancy be socially optimal. In *Section 5*, I briefly discuss how the tax base should be allocated among the governments if matching intergovernmental grants are used to eliminate the fiscal externalities associated with the taxes. *Section 6* considers extensions and concludes.

2. *A Simple Model of Optimal Tax Base Division*

I consider a model with a single state government and n local governments. Each locality has a single (representative) resident with all residents being identical. Each government finances a public service to provide to its residents with g_s being the level provided by the state government and $g_j, j=1, \dots, n$, the level provided by locality j . The public services are produced with constant costs with the cost function for the federal government service $c(g_s) = ng_s$, and the cost functions for each locality j is given by $c_j(g_j) = g_j, j=1, \dots, n$. While there are n independent local governments, each government has the same policy objectives and instruments as well as identical residents. To simplify the analysis simpler and focus on what I believe are the issues of most interest, I assume that the number of localities is large enough so that no individual locality considers the impacts its policies have on state revenues. Then in equilibrium all localities will, independently, choose the same policies. Given this symmetry, I shall refer to “local” policies denoted by the subscript l and for most of the analysis suppress notation referring to specific localities.

In addition to public services, residents consume private commodities. Following Wilson (1989), I consider a continuum of these private commodities identified on the interval $[0, 1]$. I denote the gross of tax price of commodity i , $x(i)$, by $q(i)$ with the net price of all commodities being unity. Since my interest is in how to divide the tax base between the two levels of government, I assume identical demand functions over

the set of commodities. That is, when prices are identical, the quantity demanded is the same for the commodities or more formally,

$$x(i) = x(j), \frac{\partial x(i)}{\partial q(i)} = \frac{\partial x(j)}{\partial q(j)}, \frac{\partial x(i)}{\partial q(k)} = \frac{\partial x(j)}{\partial q(k)}, k \neq i, j \text{ if } q(i) = q(j) \quad (2.1)$$

I denote these derivatives of the demand function by $x_{11} = \frac{\partial x(i)}{\partial q(i)}$ and $x_{21} = \frac{\partial x(i)}{\partial q(j)}$, $j \neq i, \forall i, j \in (0,1)$.⁵

In addition to this continuum of commodities, there is a single commodity z that is untaxed.⁶

As each local government and the state government assess commodity taxes to finance their public services, the gross price of each commodity depends on whether it is part of the local government's tax base and/or the state government's tax base. Localities also are restricted to uniform tax rates with τ_j being locality j 's tax on any commodity in the set of commodities taxed by it. The equilibrium, identical local tax rates are denoted by τ .

Localities are restricted to taxing the set of commodities on the interval $[0, \bar{k}_l]$. The set taxed by the state government is on the interval $[\bar{k}_s, 1]$. Since with co-occupancy it is possible for $\bar{k}_l > \bar{k}_s$ let the length of the interval only taxed by the local government, $[0, \min(\bar{k}_l, \bar{k}_s)] \equiv k_l$ and the length of the interval taxed only by the state government, $[\max(\bar{k}_l, \bar{k}_s), 1] \equiv k_s$. The set taxed by both governments is the interval $[\bar{k}_s, \bar{k}_l] \equiv k_b$ if $\bar{k}_l > \bar{k}_s$. The distribution of the tax base is depicted in *Figure 1*. Then the gross of tax price for the commodities in locality j can be summarized by

⁵ This, of course, should be viewed as an approximate since commodities facing different prices will not have the same demand. However, given no a priori information about the sign of $\frac{\partial^2 x(i)}{\partial q(i)q(j)}$, this approximation enables me to focus on the essential question of allocating the uniform tax base between the two levels of government. While the specific formulation of some of the results would be somewhat modified if we relaxed this assumption, the conclusions obtained, as least qualitatively, are unaffected. An example of a utility function that satisfies (2.1) is the CES function,

$$U^j = \frac{\sigma}{\sigma-1} \ln \left\{ \int_0^1 [x_j(i)]^{(\sigma-1)/\sigma} di \right\} + V^z(z_j) + V^l(g_j) + V^s(g_s)$$

⁶ The untaxed commodity z is necessary to ensure that there are distortions associated with taxes placed on the entire tax base.

$$q_j(i) = \begin{cases} 1 + \tau_j, & i \in [0, \min(\bar{k}_l, \bar{k}_s)] \\ 1 + \tau_s, & i \in [\max(\bar{k}_l, \bar{k}_s), 1] \\ 1 + \tau_j + \tau_s, & i \in [\bar{k}_s, \bar{k}_l], \bar{k}_l > \bar{k}_s \end{cases}. \quad (2.2)$$

As I assume the utility function is separable in private consumption and the two public services, the indirect utility function for a resident of locality j can be expressed as

$$V^j[q_j, g_j, g_s] = V^x(q_j) + V^s(g_s) + V^l(g_j), \quad j = 1, \dots, n \quad (2.3)$$

where I denote the sub-utility function with respect to prices by $\int_0^1 V(q(k)) dk$ and suppress the argument for \bar{x} as it is untaxed.

Let the objective function for the locality j 's government be given by

$$W^j[q_j, g_j] = V^x(q_j) + V^l(g_j), \quad (2.4)$$

Since, as discussed earlier, I assume that local governments ignore the impact of their policies on state revenues, I do not include state public services as an argument in the local government's welfare function.⁷ Analogously, for the state government I consider the objective function,

$$W^s[q_j, g_j, g_s] = \sum_{j=1}^n V^x(q_j) + nV^s(g_s) + \alpha_s \sum_{j=1}^n V^l(g_j), \quad (2.5)$$

where $\alpha_s \in [0, 1]$. If the state government is maximizing aggregate utility in the state $\alpha_s = 1$ while $\alpha_s < 1$ means a lower weight placed on local services, possibly due to voter/resident ignorance of the impacts of state decisions on local services. In the analysis that follows, I focus on two cases: 1) when the state government ignores its impacts on local revenues ($\alpha_s = 0$) and 2) when the state government maximizes aggregate utility ($\alpha_s = 1$).

Let X_j and $X_{sj}, j=1, \dots, n$ denote the local tax base and the state tax in locality j . In a symmetric equilibrium with all localities setting the same tax rate, let the tax bases be denoted by $X_s \equiv k_s x_s + k_{ls} x_{ls}$ and $X_l \equiv k_l x_l + k_{ls} x_{ls}$ where the terms x_s, x_l , and x_{ls} denotes the demand for commodities subject to only the state

⁷ In earlier versions I consider the case when the local governments take into account the impact their tax decisions have on the provision of public services to the extent that it affects the utility of the resident of that locality.

tax, only to the local tax, and to both taxes. Then the government budget constraints for the state government and local government j are given by

$$ng_s = \tau_s \sum_{j=1}^n X_{s_j} \text{ and } g_j = \tau_j X_j, \quad j = 1, \dots, n. \quad (2.6)$$

3. Externalities and the Endogenous Choice of Tax Base

In the United States, the choice of tax base, that is what local governments can tax, is not at the discretion of local government but instead determined by state governments.⁸ While this may be the case, it still may be useful to examine what tax base local governments would choose if given the option. Then I begin by considering the problem facing both local governments and the state government if they can choose both their tax rate and the extent of their base.⁹ The tax rate and base are chosen in Nash equilibrium among all local governments and the state government. Since all localities have the same objective and the state must set the same tax rates in all localities, in this equilibrium all localities will have the same tax rate and tax base. We denote the common equilibrium local tax rate and base by τ_l and \bar{k}_l .

As mentioned, the problem facing any locality j is to maximize its welfare function given the tax rates and bases of the other localities and the state government. Then formally we have

$$\underset{\tau_j, \bar{k}_j}{\text{Maximize}} W^j [q_j, g_j, g_s] = \int_0^1 V(q_j(\tau_j, \tau_s, \bar{k}_j, \bar{k}_s)) dk + V^l(g_j(\tau_j, \tau_s, \bar{k}_j, \bar{k}_s)), \quad j = 1, \dots, n \quad (3.1)$$

where $g_j(\tau_j, \tau_s, \bar{k}_j, \bar{k}_s)$ is defined by the government budget constraints, (2.6). In a symmetric equilibrium

the welfare-maximizing local tax rate and base, τ_l and \bar{k}_l , satisfy the conditions,

$$W_{\tau_l}^l = V_y \left[\underbrace{-(k_l x_l + k_{l_s} x_{l_s})}_{(a)} + \underbrace{MRS_l((k_l x_l + k_{l_s} x_{l_s}) + \tau_l (k_l + k_{l_s})(x_{11} + (k_l + k_{l_s}) x_{21}))}_{(b)} \right] = 0 \quad (3.2a)$$

and

⁸ States can and do regulate tax rates as well. State restrictions on local tax rates and bases are both examples of Dillon's Rule, the 1868 Iowa State Supreme Court opinion of John M. Dillon who wrote that municipalities were "the mere tenants at the will of the legislature" (Dillon, 1911, p. 448).

⁹ I ignore the possibility of *horizontal* fiscal externalities, that is, the possibility that changes in the tax rates in one locality affect the tax revenues collected in other localities. The implications of horizontal fiscal externalities are discussed in the concluding section.

$$(1-\bar{k}_l)W_{\bar{k}_l}^l = (1-\bar{k}_l)V_y \tau_l [(MRS_l - 1)x_z + MRS_l \tau_l (k_l + k_{ls})x_{21}] = 0, \quad \begin{array}{l} z=ls \text{ if } \bar{k}_l \geq \bar{k}_s \\ z=l \text{ if } \bar{k}_l < \bar{k}_s \end{array} \quad (3.2b)$$

where $MRS_j = \frac{\partial V}{\partial g_j} / \frac{\partial V}{\partial y}$, $j = l, s$. Derivation of (3.2) is found in the *Appendix*. Critical to understanding the policy choices of both governments and the optimality of these decisions is how changes in tax rates and tax bases affect tax revenues. Derivations of the impacts of changes in the tax rates and tax bases on tax revenues are also found in the *Appendix*. Note that in (3.2b), given the discrete change in the price of $x(\bar{k}_l)$ I use the first order approximation of $\int_0^{\bar{k}} \frac{\partial V(q(k))}{\partial \alpha(k)} dk \tau_l = -V_y x(\bar{k}_l) \tau_l$. The impact on the tax bases from an expansion of the tax base also depends on whether there is an existing overlap ($\bar{k}_l > \bar{k}_s$) or not. If there is overlap then the addition of \bar{k}_l directly adds x_{ls} to the local base but reduces the state tax base by reducing $x(\bar{k}_l)$ from x_s to x_{ls} a decrease of $\tau_l x_{1l}$.

Analogous to the local governments, the state government sets its tax rate and base to maximize its welfare function. Formally, the problem the state government is facing is

$$\underset{\tau_s, \bar{k}_s}{\text{Maximize}} W^s = \sum_{j=1}^n V^x(q_j) + nV^s(g_s(\tau_1, \dots, \tau_n, \bar{k}_1, \dots, \bar{k}_n, \tau_s, \bar{k}_s)) + \alpha_s \sum_{j=1}^n V^l(g_j(\tau_j, \bar{k}_j, \tau_s, \bar{k}_s)) \quad (3.3)$$

Then in the Nash equilibrium, the state's tax rate and base must satisfy the first order conditions,

$$W_{\tau_s}^s = V_y \left[\begin{array}{l} -(k_s x_s + k_{ls} x_{ls}) + \alpha_s MRS_l \tau_l (k_{ls} x_{11} + (k_s + k_{ls})(k_l + k_{ls})x_{21}) \\ + MRS_s ((k_s x_s + k_{ls} x_{ls}) + \tau_s (k_s + k_{ls})(x_{11} + (k_s + k_{ls})x_{21})) \end{array} \right] = 0 \quad (3.4a)$$

and

$$\bar{k}_s W_{\bar{k}_s}^s = -\bar{k}_s \tau_s V_y \left[\begin{array}{l} (MRS_s - 1)x_z + MRS_s \tau_s (k_s + k_{ls})x_{21} \\ + \alpha_s MRS_l \tau_l (D \bullet x_{11} + (k_l + k_{ls})x_{21}) \end{array} \right] = 0, \quad \begin{array}{l} z=ls \text{ and } D=1, \text{ if } \bar{k}_l \geq \bar{k}_s \\ z=s \text{ and } D=0, \text{ if } \bar{k}_l < \bar{k}_s \end{array} \quad (3.4b)$$

3.1 Externalities and the Choice of Taxes

While the tax rate that satisfies (3.2) maximizes the utility of each locality's residents given the policies of the $n-1$ other localities and the state government, these policies, in general, do not maximize social welfare. This is because a locality's tax rates affects state tax revenue and therefore the level of the state service for the residents of the $n-1$ other localities. Analogously, if the state government does not fully weigh the local services in its welfare function ($\alpha_s < 1$), it, too, will also generate a fiscal externality. To determine the fiscal ex-

ternality from a locality's tax, differentiate aggregate utility $\left(W = \sum_{i=1}^n V^i\right)$ with respect to the tax rate of a single locality j (τ_j) and evaluate at the equilibrium local tax rate of τ_l to obtain

$$W_{\tau_j} = \left[-V_y X_j + V_j^j \left(X_j + \tau_j \frac{\partial X_j}{\partial \tau_j} \right) \right] + \left(V_{g_s}^j + \sum_{\substack{i=1 \\ i \neq j}}^n V_{g_s}^i \right) \left(\frac{\tau_s}{n} \frac{\partial X_{s_j}}{\partial \tau_j} \right), j=1, \dots, n. \quad (3.5)$$

using $g_s = \frac{1}{n} \tau_s \sum_{j=1}^n X_{s_j}$. At the equilibrium rate of τ_l term (a) of (3.5) equals zero by the envelope theorem.

Then dividing (3.5) by $-V_y \frac{\partial [\tau_l X_l]}{\partial \tau_l}$ to express it in terms of marginal costs of funds gives

$$EMCF_l = MRS_s \tau_s \left(k_{ls} x_{11} + (k_l + k_{ls})(k_s + k_{ls}) x_{21} \right) D_l^{-1} \quad (3.6a)$$

where $D_l > 0$.¹⁰ $EMCF_l$ is the *external marginal cost of funds* associated with an increase in local spending. Intuitively, the external marginal cost of funds depends on the product of the marginal rate of substitution for the state service and the change in state revenue, the product of the state tax rate and the change in the state tax base. The number of localities and the weight placed by local governments on state services matter as well as they determine how much of the impact of local policies is on state services are “internalized” into the local tax decision. The external marginal cost of funds for the state is analogously to that of the local governments and is given by

$$EMCF_s = (1 - \alpha_s) MRS_l \tau_l \left(k_{ls} x_{11} + (k_s + k_{ls})(k_l + k_{ls}) x_{21} \right) D_s^{-1} \quad (3.6b)$$

with $D_s > 0$ and, of course, the number of state government units equaling one. From (3.6) we obtain several insights into vertical externalities:

Proposition 1. a) Assume that $\alpha_s \neq 1$. Then:

- i) $Sign\{EMCF_l\} = Sign\{EMCF_s\};$
- ii) $EMCF_j > (<) 0$ if $-x_{11} > (<) \frac{[(k_l + k_{ls})(k_s + k_{ls})]}{k_{ls}} x_{21}, j = l, s;$ (3.7)
- iii) *If the commodities are not gross substitutes ($x_{12} \leq 0$) then $EMCF_j > 0, j=l, s;$*
- iv) *If the tax bases for the local government and the state government are identical then $EMCF_j > 0, j = l, s.$*
- b) *If the state maximizes aggregate utility ($\alpha_s = 1$) then $EMCF_s = 0.$*

¹⁰The term $D_l = (k_l x_l + k_{ls} x_{ls}) + \tau_l (k_l + k_{ls}) (x_{11} + (k_l + k_{ls}) x_{21}) > 0$; D_s is defined analogously.

Part a) of the proposition indicates symmetry in the externalities. The signs of the state and local externalities are always the same, though not necessarily negative as *part ii)* indicates. If commodities are gross complements, an increase in the price of any commodity reduces the demand for all other commodities, thereby decreasing both tax bases; if commodities are gross substitutes, an increase in the local (state) tax rate on the overlapping base will decrease the demand for commodities in the overlapping base thereby reducing the state (local) tax base. However, increases in the local (state) tax base increase the part of the state (local) tax base that does not overlap. Thus, the sign of $EMCF_j, j=l,s$ depends on two distinct factors – the own price elasticity relative to the cross-price elasticities and the extent of the overlap of the tax base relative to the extent that the tax bases are independent. They will, however, both be negative if both governments tax the entire base. As *Part b)* indicates there is no externality associated with the state’s tax rate if it chooses its tax rate to maximize aggregate utility. In this case it fully internalizes the impact of its tax policies on local services. In contrast, as long as there is more than one locality, there will be fiscal externalities associated with local tax

decisions, provided $-x_{11} \neq \frac{[(k_l + k_s)(k_s + k_s)]}{k_s} x_{21}$.

In *Table 2* I use (3.7) to determine the number (length of interval) of overlapping commodities at which $EMCF_j$ is equal to zero. I normalize the total number of taxable commodities to be 100 and exogenously choose the number of commodities in the local tax base to be 25, 50, or 75. In addition, I allow the own-price elasticity of demand to vary as well. From the consumer’s budget constraint and symmetry we have the condition $(K - 1)\epsilon_{21} = -(1 + \epsilon_{11})$ where ϵ_{ij} is the elasticity of demand for good i with respect to the price of good j to define the cross price elasticity and K is the number of commodities.¹¹ The results of this numerical example suggest that the amount of possible overlap can be potentially quite large with the extent of the overlap increasing with the magnitude of the own-price elasticity of demand. This, of course, is because the

greater (more negative) the own-price elasticity the stronger the substitutes are the commodities in the tax base. Note there is not an obvious simple relationship between overlap and the extent of the local tax base.

3.2 The Choice of Tax Base

To characterize the choice of tax base for the local governments, first evaluate $W_{\bar{k}_l}^l$ (3.2b) at $\bar{k}_l=0$ using the first order condition with respect to the tax rate (3.2a) to simplify.¹² This gives

$$W_{\bar{k}_l}^l \Big|_{\bar{k}_l=0} = -V_y \tau_l^2 MRS_l x_{11} > 0. \quad (3.8a)$$

From (3.8a) it is apparent that the local government will always choose to tax a commodity not taxed by the state government. In equilibrium, then, no commodity will be untaxed. Evaluating (3.2b) with $\bar{k}_l > 0$, again with the use of (3.2a), gives

$$W_{\bar{k}_l}^l \Big|_{\bar{k}_l>0} = \frac{V_y \tau_l (-x_{11})}{(k_l + k_{ls})} \left[MRS_l (k_l (\tau_l - \tau_s) + k_{ls} \tau_l) + k_l \tau_s \right]. \quad (3.8b)$$

The sign of (3.8b) is almost certainly positive as it reflects the impacts on local revenues and the impact on the price of $x(\bar{k}_l)$.

Then, from examination of the tax base chosen by the localities, (3.8), we know that in equilibrium there will be some co-occupancy of the tax base with localities choosing to tax the entire base.

Then we can focus on the state's choice of tax base given $\bar{k}_l > 0$. Evaluating $W_{\bar{k}_s}^s$ with $\bar{k}_l > 0$ and using (3.4a) to simplify gives

$$W_{\bar{k}_s}^s \Big|_{\bar{k}_l>0} = \frac{\tau_s (-x_{11}) V_y}{(k_l + k_{ls})} \left[MRS_s \left(k_s (\tau_s - \tau_l) + k_{ls} \tau_s \right) + k_s \tau_l - \alpha_s k_s \tau_l MRS_l \right]. \quad (3.9)$$

Term (a), the impacts of the increase in the state tax base on state public services and the price of $x(\bar{k}_s)$ on state welfare is positive. However, the impact on local services (term (b)) is negative, reducing the increases in

¹¹ Using a discrete version, we have $\sum_{j=1}^K \frac{\partial [q(j)x(j)]}{\partial q(i)} = x(i) + \sum_{j \neq i} q(j) \frac{\partial x(j)}{\partial q(i)} + q(i) \frac{\partial x(i)}{\partial q(i)} = 0$ which gives

$$\sum_{j \neq i} \frac{q(i)}{x(j)} \frac{\partial x(j)}{\partial q(i)} \frac{q(j)x(j)}{q(i)x(i)} = - \left(1 + \frac{q(i)}{x(i)} \frac{\partial x(i)}{\partial q(i)} \right). \text{When } q(i)=q(j) \text{ then } x(i)=x(j) \text{ and we obtain } (K-1) \varepsilon_{21} = -(1+\varepsilon_{11}).$$

state welfare from expanding its tax base. Then given the weight (α_s) the state places on local public services, the sign of (3.9) appears ambiguous. However from (3.8b), we know that k_s will equal zero (local governments will tax the entire base). Then in (3.9) term (b) equals zero and term (a) reduces to $k_{ls}MRS_s\tau_s$ making $W_{k_s}^s > 0$.

Proposition 2: Assume both the local and state governments independently choose their tax bases.

- a) Then the equilibrium tax bases are such that both levels of government tax the entire tax base, that is, $\bar{k}_l = 1$ and $\bar{k}_s = 0$.
- b) Assume $\alpha_s = 1$. In equilibrium, $MRS_s > MRS_l$.

Regardless of the valuation of local public services by the state government and whether its tax policies create any fiscal externalities, with many localities, both levels of governments will choose to tax the entire base and thereby have identical tax bases. Then from *Proposition 1* if $\alpha_s \neq 1$ with identical tax bases both fiscal externalities will be negative, meaning that the equilibrium tax rates exceed the social welfare maximizing rates. As shown in the *Appendix*, if $\alpha_s = 1$, that is, the state sets its policies to maximize aggregate utility, the marginal rate of substitution for the state public service will exceed that of the local public service – the local public service, relative to the state service, is overprovided.

Finally, as with the tax rate, we can examine the impact of the expansion of the tax base on social welfare. When there is no co-occupancy, $k_{ls} = 0$, the impact on social welfare of an increase in the local tax base is given by

$$W_{k_l} = V_y \tau_l \left[\underbrace{\tau_l MRS_l}_{(a)} (-x_{11}) + \underbrace{MRS_s \tau_s k_s}_{(b)} x_{21} \right] \quad (3.10)$$

where $W = \sum_{j=1}^N V_j (\tau_j, \tau_s, g_j(\tau_j, \tau_s), g_s(\tau_j, \tau_{-j}, \tau_s))$. Term (a) of (3.10) is the impact on the welfare of locality j and term (b) is the impact on utility in all localities due to the change in the state public service. If commodities are substitutes ($x_{21} > 0$), then (3.10) is clearly positive as expanding the local tax base not only reduces the marginal cost of funds for local services but also reduces it for the state funds by increasing the demand for commodities in the state tax base. If, commodities are complements ($x_{21} < 0$) then expansions in the local tax base act to increase the marginal cost of funds for the state government by reducing demand for the commo-

¹² See Appendix for derivation of (3.8) and (3.9).

ditions in its tax base. The impacts with co-occupancy are summarized in *Proposition 3* below.

Proposition 3.

a) Assume that $\alpha_s = 1$ with $\bar{k}_l^* = 1$ and $\bar{k}_s^* = 0$. Then:

$$W_{\bar{k}_l}^- = V_y \tau_l [(MRS_l - MRS_s)X + MRS_l \tau_l (-x_{11})]$$

$$i) \quad = V_y \tau_l \left[\frac{[\tau_s (x_{11} + x_{21})]X^2}{D} + MRS_l \tau_l (-x_{11}) \right] \quad (3.11a)$$

$$ii) \quad W_{\bar{k}_s}^- = -V_y \tau_s^2 MRS_s (-x_{11}) < 0. \quad (3.11b)$$

b) Assume that with $\alpha_s = 0$, $\bar{k}_l^* = 1$ and $\bar{k}_s^* = 0$. Then:

$$W_{\bar{k}_l}^- = V_y \tau_l [(MRS_l - MRS_s)X + MRS_l \tau_l x_{21}]$$

$$= V_y \tau_l \left[\frac{(\tau_s - \tau_l)}{D} (x_{11} + x_{21})X^2 + MRS_l \tau_l x_{21} \right] \quad (3.12a)$$

and

$$W_{\bar{k}_s}^- = -V_y \tau_s [(MRS_s - MRS_l)X + MRS_s \tau_s x_{21}]$$

$$= -V_y \tau_s \left[\frac{(\tau_l - \tau_s)}{D} (x_{11} + x_{21})X^2 + MRS_s \tau_s x_{21} \right] \quad (3.12b)$$

When both levels of government tax the entire base, there is a tradeoff between the gain from expanding the base for one government and the reduction in the revenue for the other level of government. We provided two expressions of the impact of an increase of the local tax base. There is a gain from expanding the local tax base, a reduction in the marginal cost of funds for local taxes ($-MRS_l \tau_l x_{11}$). The direct impact, $MRS_l - MRS_s$ is, as shown in *Proposition 2*, negative, making the impact of an increase in the local tax base ambiguous. *Part a.ii)* simply states that if the state government is choosing policies to maximize social welfare ($\alpha_s = 1$) then given the local government taxes the entire base it is clearly welfare maximizing for the state to do so as well. Finally, consider the case when $\alpha_s = 0$. As (3.12) suggests, it is possible for either $W_{\bar{k}_l}^- > 0$ or $W_{\bar{k}_s}^- < 0$ with the possibility that both conditions hold, that is, that a decrease in either tax base would reduce utility. As our expressions for $W_{\bar{k}_l}^-$ and $W_{\bar{k}_s}^-$ suggest, reducing the tax base of either level of government would reduce social welfare if the marginal rates of substitution or, equivalently in this case, the tax rates are approximately equal and commodities are substitutes.

An implication of *Proposition 3* is that if both governments are taxing the entire base, it is not necessarily welfare improving to (marginally) reduce the tax base of one or both of the levels of government. For

the case in which $\alpha_s = 1$ and $x_{2l} > 0$ then a decrease in the local tax base may reduce welfare if MRS_l is not significantly below MRS_s . When $\alpha_s = 0$, if $MRS_s \approx MRS_l$, ($\tau_l \approx \tau_s$) and $x_{2l} > 0$ then a decrease in either tax base would decrease welfare. Essentially we would be moving from an equilibrium in which all commodities are taxed equally to one in which there is not uniform taxation and, as a result, one in which MRS_l and MRS_s are no longer equal.

4. *Optimal Tax Base Division and Co-Occupancy*

When given the option with a large number of local governments, both levels of government choose to tax the entire tax base. Reductions in the extent of one government's tax base may or may not be socially optimal given the tax base chosen by the other government. Here I address the question of the social-welfare maximizing division of the tax base as well as whether co-occupancy is optimal. I first address the question of how the tax base should be divided between the two levels of government if there is to be no co-occupancy. After deriving the optimal division of the tax base, the question of whether the state and local governments should share tax bases, that is, whether there should be any overlap in the two tax bases, is then addressed.

Before formally examining the problem of how to divide the tax base between the two levels of government in the absence of co-occupancy, consider what would characterize the "ideal" division of the tax base. This division would yield what might be considered the "second-best" outcome, the outcome achieved when a single government (state) finances both services and can set any tax rate it desires on the commodities. Given our simple model in which all commodities have the same own-price and cross-price elasticities, if possible, the tax rates levied by the two levels of government should be equal. Since the marginal cost is the same for the two public services, their marginal rates of substitution (with respect to the private commodities) should also be equal. However, the agency or government determining the division of the tax base does so without any control over how the state and local governments set their tax rates given the division of the tax base. As will be shown, the "ideal" outcomes of $\tau_l = \tau_s$ and $MRS_l = MRS_s$ are generally not both obtainable. Thus, in the absence of co-occupancy, this ideal outcome is generally not possible, suggests that the optimality of the overlap of the two tax bases should be not be dismissed immediately.

4.1 The Optimal Division of the Tax Base in the Absence of Co-occupancy

Continuing with the same notation as used in Section 3, in the absence of co-occupancy we have a state tax base of $\int_{k_l}^1 x(k)dk$ and a local tax base of $\int_0^{\bar{k}_l} x(k)dk$. Given the tax rates chosen by the state government, τ_s , and the local government, τ_l , the problem facing a social planner determining the optimal division of the tax base between the two levels of government is

$$\underset{\bar{k}_l}{\text{Maximize}} W^P = \int_0^1 v(q(k))dk + V^s(g_s(\tau_s, \tau_l, \bar{k}_l)) + V^l(g_l(\tau_s, \tau_l, \bar{k}_l)) \quad (4.1)$$

where the social planner, by choosing the extent of the local tax base, determines the extent of the state tax base since $\bar{k}_l = \bar{k}_s$. As an alternative to the Nash equilibrium I consider here, the social planner could be a Stackelberg leader choosing the division of the tax base first. In that case, the social planner chooses the division of the tax base considering the impact of its choice on the tax rates chosen by the state and local governments. Since the results of the two approaches appear to be very similar in their implications for tax policy, I chose to focus on the simpler Nash equilibrium.¹³ The first order condition for (4.1) in the symmetric equilibrium can be expressed as

$$W_{\bar{k}_l}^P = V_y \left[(MRS_l - 1)\tau_l x_l - (MRS_s - 1)\tau_s x_s + (MRS_l \tau_l k_l + MRS_s \tau_s k_s) x_{21} (\tau_l - \tau_s) \right] = 0. \quad (4.2)$$

To better understand (4.2) recall that the impact of a marginal increase in the local tax base is given by $\frac{\partial \int_0^{\bar{k}_s} x(i)di}{\partial \bar{k}_l} = x_l + k_l x_{21} (\tau_l - \tau_s)$. The addition of $x(\bar{k}_l)$ to the local tax base and its removal from the state base affects the local tax base by the direct increase in the base by adding $x(\bar{k}_l) = x_l$ and by the impact of the change of the tax rate on $x(\bar{k}_l)$ on the demand for the other commodities in the local tax base. Analogously, the marginal decrease in the state tax base is $-x_s + k_s x_{21} (\tau_l - \tau_s)$. Finally, there is the change in utility associated with the change in the price of commodity of $x(\bar{k}_l)$. The first order approximation to this discrete

¹³ In the Stackelberg equilibrium the first order condition for (4.1) is

$$W_{\bar{k}_l}^P = V_y \left[(MRS_l - 1)\tau_l x_l - (MRS_s - 1)\tau_s x_s + (MRS_l \tau_l k_l + MRS_s \tau_s k_s) x_{21} (\tau_l - \tau_s) \right] + \left(1 - \frac{\alpha_l}{n} \right) MRS_s k_s x_{21} \tau_l' + (1 - \alpha_s) MRS_l k_l x_{21} \tau_s' = 0 \quad \text{where } \tau_j' = \frac{\partial \tau_j}{\partial \bar{k}_l}, j = l, s.$$

change in price of $(\tau_l - \tau_s)$ is $-V_y(\tau_l x_l - \tau_s x_s)$. Using the first order conditions for state and local tax rates,

(3.2a) and (3.4a) in (4.2) gives

$$W_{k_l}^P = V_y \left[-MRS_l^s \tau_l [\tau_l x_{l1} + (1 - \alpha_s) \tau_s k_l x_{21}] + MRS_s^s \tau_s [\tau_s x_{l1} + \tau_l k_s x_{21}] \right] = 0. \quad (4.3)$$

To better understand the implications of (4.2) and (4.3) for the division of the tax base between the two levels

of governments as well as the tax rates they set let $\tau_s(\bar{k}_l)$ and $\tau_l(\bar{k}_l)$ denote the tax rates chosen by the

state and local governments for \bar{k}_l . Further assume that $\tau_s'(\bar{k}_l) > 0$ and $\tau_l'(\bar{k}_l) < 0$, that decreasing the

state tax base will increase the state tax rate and increasing the local tax base will decrease the local tax rate.

Then let τ_j^p , $j = l, s$ and MRS_j^s , $j = l, s$ denote the tax rate and marginal rate of substitution for govern-

ment j at the division of the tax base that satisfies (4.3), \bar{k}_l^p . Then it follows that:

Proposition 4. a) If and only if $x_{21} = 0$ or if $\tau_l \left(\frac{1}{2 - \alpha_s} \right) = \tau_s \left(\frac{1}{2 - \alpha_s} \right)$ can the conditions $\tau_l^p = \tau_s^p$ and $MRS_l^p = MRS_s^p$ be simultaneously satisfied.

b) If the state maximizes aggregate utility ($\alpha_s = 1$) and $x_{21} \neq 0$, then the conditions $\tau_l^p = \tau_s^p$ and $MRS_l^p = MRS_s^p$ cannot be simultaneously satisfied.

Proof of Proposition 4 is found in the Appendix. If $x_{21} = 0$, there are no externalities generated by either government when there is no co-occupancy and the marginal cost of funds (MCF) depends only on the tax rates and

not the division of the tax base. Then for any division of the tax base in which $\tau_l^p = \tau_s^p$, it follows that

$MRS_l^p = MRS_s^p$. However, if $x_{21} \neq 0$, then the division of the tax base does affect the marginal cost of

funds and externalities are also present. In this case only if $\tau_l^p = \tau_s^p$ at $\frac{1}{2 - \alpha_s}$ will both conditions simultane-

ously be satisfied. Note that when the state government is maximizing aggregate utility ($\alpha_s = 1$) then there is

no division of the tax base for which both conditions can be satisfied.

Proposition 4 suggests that the conditions $\tau_l^p = \tau_s^p$ and $MRS_l^p = MRS_s^p$ are unlikely to be obtained through division of the tax base alone if $x_{21} \neq 0$. What is less obvious is how the optimal division of the tax base might, in fact, be characterized. When $k_b = 0$, using the first order conditions for the tax rates, (3.2a) and

(3.4a) we obtain

$$MRS_l - MRS_s = \frac{(\tau_l - \tau_s)(-\tilde{x}_{11}) + (\tau_s k_s - \tau_l k_l (1 - \alpha_s)) \tilde{x}_{21}}{\tilde{D}}. \quad (4.4)$$

where $\tilde{x}_{11} = \frac{x_{11}}{x}$ and $\tilde{x}_{21} = \frac{x_{21}}{x}$.¹⁴ Our interest is considering the relationship between the two tax rates and the marginal rates of substitution for the two public services when the tax base is evenly split ($\bar{k}_l = .5$) to better understand what the optimal split of the tax base might be. At $\bar{k}_l = .5$ we can express (4.4) as

$$MRS_l - MRS_s = \frac{(\tau_l - \tau_s)(-\tilde{x}_{11} + .5(-\tilde{x}_{21})) + .5\alpha_s \tau_l \tilde{x}_{21}}{\tilde{D}} \quad (4.5)$$

When $\alpha_s = 0$, the sign of $MRS_l - MRS_s$ is the sign of $\tau_l - \tau_s$, the level of government with the higher tax rate also has its public service relatively underprovided. From (4.3) it is relatively easy to see that $W_{k_l}^P|_{\bar{k}_l=.5} > (<) 0$ if $\tau_l(.5) > (<) \tau_s(.5)$ and the optimal tax base, \bar{k}_l^P , will be greater or less than .5, depending on whether $\tau_l(.5) > (<) \tau_s(.5)$. If $\alpha_s = 1$, our other case of interest, the relationship between the tax rates and marginal rates of substitution is less clear. If commodities are substitutes ($\tilde{x}_{21} > 0$), if $\tau_l > \tau_s$ implies $MRS_l > MRS_s$ but $\tau_s > \tau_l$ does not imply that $MRS_s > MRS_l$. Conversely, when the commodities are complements ($\tilde{x}_{21} < 0$), if $\tau_s > \tau_l$, $MRS_s > MRS_l$ but $\tau_l > \tau_s$ does not imply that $MRS_s > MRS_l$. Again using (4.3) we can see that when $\tilde{x}_{21} > 0$ and $\tau_l > \tau_s$, $W_{k_l}^P|_{\bar{k}_l=.5} > 0$ while $W_{k_l}^P|_{\bar{k}_l=.5} < 0$ when $\tilde{x}_{21} < 0$ and $\tau_s > \tau_l$. Summarizing these results and some of their implications, we have

Proposition 5.

- a) Assume that $\alpha_s = 0$ then if $\tau_l(.5) > (<) \tau_s(.5)$:
 - i) The division of the tax base that satisfies (4.3), \bar{k}_l^P , must be such that $\bar{k}_l^P > (<) .5$.
 - ii) If $x_{21} = 0$ then $\tau_l(\bar{k}_l^P) = \tau_s(\bar{k}_l^P)$; b) If $x_{21} > 0$, $\tau_l(\bar{k}_l^P) > (<) \tau_s(\bar{k}_l^P)$; c) If $x_{21} < 0$, $\tau_l(\bar{k}_l^P) < (>) \tau_s(\bar{k}_l^P)$
 - iii) Assume $x_{21} \neq 0$. If $\tau_l(\bar{k}_l^P) > (<) \tau_s(\bar{k}_l^P)$ then $MRS_l < (>) MRS_s$.
- b) Assume that $\alpha_s = 1$:
 - i) If $\tau_l(.5) > \tau_s(.5)$ and $\tilde{x}_{21} > 0$ then $\bar{k}_l^P > .5$, $\tau_l(\bar{k}_l^P) > \tau_s(\bar{k}_l^P)$, and $MRS_l < MRS_s$.

¹⁴The expressions $\tilde{x}_{21} = \frac{x_{21}}{x}$ and $\tilde{x}_{11} = \frac{x_{11}}{x}$ where we make the approximation $x_l \approx x_s = x$. The term $\tilde{D} = \tilde{D}_l \tilde{D}_s > 0$ where $\tilde{D}_j = 1 + \tau_j [\tilde{x}_{11} + k_j \tilde{x}_{21}]$, $j=1,2$.

ii) If $\tau_L(.5) < \tau_S(.5)$ and $\tilde{x}_{21} < 0$ then $\bar{k}_l^p < .5$, $\tau_l(\bar{k}_l^p) < \tau_s(\bar{k}_l^p)$ and $MRS_l > MRS_s$.

Proof of *Proposition 5* is found in the appendix. For the cases for which we can determine the optimal division of the tax base, the results with $\alpha_s = 1$ follow those found with $\alpha_s = 0$. Essentially, if it is the case that when the tax base is evenly divided ($\bar{k}_l = .5$) and we either have: a) $\tau_L(.5) > \tau_S(.5)$ and $MRS_l > MRS_s$ or b) $\tau_L(.5) < \tau_S(.5)$ and $MRS_l < MRS_s$, we increase the tax base for the level of government with the higher tax rates and marginal rate of substitution for its public service. With $x_{21} \neq 0$ (or the case discuss in *Proposition 4*), the optimal division of the tax base only can occur when $\tau_l(\bar{k}_l^p) > \tau_s(\bar{k}_l^p)$, and $MRS_l < MRS_s$ or $\tau_l(\bar{k}_l^p) < \tau_s(\bar{k}_l^p)$, and $MRS_l > MRS_s$ as it will never be optimal to have both a higher tax rate and relative under-provision of a public service by one of the two levels of government. In *Table 3* we provide the results of some numerical simulations in which equation (4.3) was examined using a very simply parameterization. The number (length) of commodities is set at 100. Income is 140. Public services are chosen so that the sum of demand, in the absence of distorting taxes is 40 with the allocation for the two governments varying through the course of the exercise. In the absence of taxes, one unit of each commodity is consumed. Three own-price elasticities were reported, -1, -.5 and -2 with consistent cross-price elasticities. We undertake simulations for both the case of $a_i = 0$ as well as the case of $\alpha_s = 1$. The optimal division of the tax base, as well as the tax rates and marginal rates of substitution are reported for each of the simulations.

Finally, we might consider whether the elimination of co-occupancy of the tax base leads to social-welfare maximizing tax rates for the two levels of government. Then differentiating the social welfare function with respect to the tax rate for a local government gives

$$W_{\tau_l} = \frac{1}{n} V_{g_s} \tau_s \bar{k}_s \bar{k}_l x_{21}. \quad (4.6a)$$

Differentiating the social welfare function with respect to state tax rate gives

$$W_{\tau_s} = (1 - \alpha_s) V_{g_l} \tau_l \bar{k}_s \bar{k}_l x_{21}. \quad (4.6b)$$

The impact on social welfare depends on the change in the tax revenue of the other level of govern-

ment and the extent to which this impact has been incorporated into the choice of taxes by the level of government. An increase in revenue will increase social welfare; a decrease in revenue will decrease social welfare. Then it follows that:

Proposition 6: Assume that the tax base is optimally divided between the two governments. Then at the optimal division:

- a) *If $x_{21} = 0$, (marginal) changes in the tax rates of either government have no impact on social welfare;*
- b) *if $x_{21} > 0$, an increase in the tax rates of both or either government will increase social welfare;*
- c) *if $x_{21} < 0$, a decrease in the tax rates of both or either government will increase social welfare.*

Only if the cross-price elasticities between commodities equal zero are the tax rates welfare maximizing. This is because elimination of the co-occupancy eliminates the fiscal externality in this case. With non-zero cross-price elasticities, elimination of co-occupancy does not eliminate the fiscal externality. In the case of gross substitutes, it may change the fiscal externality from being negative with co-occupancy to being positive with no co-occupancy. This, in turn, means that taxes also change from being “too” high to being “too” low, below the welfare maximizing rates. Regardless, elimination of co-occupancy, even with the optimal division of the tax base does not eliminate fiscal externalities associated with the tax rates set by the two levels of government if the governments do not internalize the externalities themselves when choosing their tax rates.

4.2 The Optimal Co-Occupancy of Tax Bases with Independent Governments

That, in general, tax rates and marginal rates of substitutions for the public services are not equal and tax increases or decreases can enhance social welfare suggests the possibility that co-occupancy could be desirable. To formally determine if co-occupancy is desirable, consider the problem of determining the optimal (social-welfare maximizing) local tax base given the extent of the state tax base,

$$\text{Maximize } V^P(\tau_s(k_s, k_l), \tau_s(k_s, k_l)) = \int_0^1 v(q(k)) dk + V_s(g_s(\tau_s(k_s), k_s)) + V_l(g_l(\tau_l(k_s), k_s)) \quad (4.7)$$

The distinction between (4.7) and (4.1) is that here \bar{k}_s is fixed and $\bar{k}_l = \bar{k}_s$ is not a necessary condition.

Then the first order condition for (4.7) is

$$w_{k_l}^P = \int \frac{\partial v(q(k))}{\partial q(k_s)} dk \tau_l + V_{g_s} \tau_s \left(x_s - x_{ls} + \int_0^{\bar{k}_s} \frac{\partial x(k)}{\partial q(\bar{k}_s^*)} dk \tau_l \right) + V_{g_l} \tau_l \left(x_{ls} + \int_{\bar{k}_s}^1 \frac{\partial x(k)}{\partial q(\bar{k}_s)} dk \tau_l \right) = 0 \quad (4.8)$$

Simplifying (4.7) similarly to (4.2) using $\tau_l x_{l1} \approx x_s - x_{ls}$ and $x_l + \tau_l x_{ll} \approx x_{ls}$ gives

$$W_{k_l}^P = V_y \left[\underbrace{[(MRS_l - 1)\tau_l x_l + (MRS_l \tau_l (k_l + k_{ls}) + MRS_s \tau_s (k_s + k_{ls}))\tau_l x_{21}]}_{(a)} + \underbrace{[MRS_s + MRS_l]\tau_l \tau_s x_{11}}_{(b)} \right] \quad (4.9a)$$

Analogously for an increase in the state tax base (decrease in \bar{k}_l) we have

$$-W_{k_s}^P = V_y \left[\underbrace{[(MRS_s - 1)\tau_s x_s + (MRS_l \tau_l (k_l + k_{ls}) + MRS_s \tau_s (k_s + k_{ls}))\tau_s x_{21}]}_{(a)} + \underbrace{[MRS_s + MRS_l]\tau_l \tau_s x_{11}}_{(b)} \right] \quad (4.9b)$$

Term (a) in both (4.9a) and (4.9b) gives the increase in welfare from expanding the tax base to an untaxed commodity in (4.7a) an expansion of the local tax base and in (4.7b) an expansion of the state tax base. Term (b) in both expressions gives the impact of expanding the overlap in tax bases on revenue.

Our interest is in the impact of an expansion of the tax base of either level of government when there is no co-occupancy ($k_b=0$) and the tax base is optimal divided between the two governments. Given an optimal division of the tax base, term (a) in (4.9a) equals term (a) in (4.9b) since the condition describing the optimal division of the tax base, (4.2), is simply term (a) from (4.9b) subtracted from term (a) in (4.9a). Then given term (b) is the same in both equations, it must be the case that if an increase in the local tax base will increase social welfare when the tax base is optimally divided, so must an increase in the state tax base. Using the first order condition for the local tax, (3.2a), in (4.9a) and evaluating at $k_b = 0$ we can express $W_{k_l}^P$ as:

$$W_{k_l}^P \Big|_{k_b=0} = \tau_l V_y \left[\underbrace{MRS_s \tau_s (x_{11} + k_s x_{21})}_{(a)} + \underbrace{MRS_l (\tau_s - \tau_l) x_{11}}_{(b)} \right] \quad (4.10a)$$

Using the first order conditions for the state taxes, (3.4a) with $\alpha_s = 0$ in (4.9b) gives

$$-W_{k_s}^P \Big|_{k_b=0} = \tau_s V_y \left[\underbrace{MRS_l \tau_l (x_{11} + k_l x_{21})}_{(a)} + \underbrace{MRS_s (\tau_l - \tau_s) x_{11}}_{(b)} \right] \quad (4.10b)$$

Term (a) of both (4.10a) and (4.10b) are both negative. While term (b) of (4.10a) is positive if $\tau_l > \tau_s$ term (b) of (4.10b) will be negative in this case; if $\tau_s > \tau_l$, term (b) of (4.10b) will be positive but term (b) of (4.10a) must be negative. Then it follows that both (4.10a) and (4.10b) cannot both be positive and therefore co-occupancy, through increases in either tax base cannot be optimal when (4.2) is satisfied. If $\alpha_s = 1$, we have

$$-W_{k_s}^P \Big|_{k_b=0} = \tau_s V_y \left[\underbrace{MRS_l \tau_l x_{11}}_{(a)} + \underbrace{MRS_s (\tau_l - \tau_s) x_{11}}_{(b)} \right]. \quad (4.10b')$$

Then, as with the case of $\alpha_s = 0$, clearly we cannot have both (4.10a) and (4.10b') both be positive. Sum-

marizing:

Proposition 7: Co-occupancy of the tax base is never optimal if the tax base is optimally divided in the absence of co-occupancy, that is, if the division of tax base satisfies (4.2).

Of course, if the tax base is not optimally divided, specifically if it is the case that $\tau_l > \tau_s$ and $MRS_l > MRS_s$, or $\tau_s > \tau_l$ and $MRS_s > MRS_l$, then co-occupancy could be welfare-improving. One of the most interesting aspects of this result is that it is true regardless of the cross-price elasticities of the commodities and, consequently, the extent and direction of the vertical fiscal externality arising from tax increases. Of course, if $x_{2l} = 0$, there is no vertical fiscal externality in the absence of co-occupancy and if $x_{2l} < 0$ having or increasing the overlap in the two tax bases only serves to increase the negative fiscal externality, so the result is not unexpected in either of these two cases. However, if $x_{2l} > 0$, then in the absence of any overlap a positive fiscal externality exists. Then co-occupancy could, in fact, eliminate any fiscal externality as discussed in *Section 3*. This, too, is not optimal. Then while arguments for eliminating co-occupancy generally seem premised on the notion of eliminating a fiscal externality associated with the tax rates, eliminating co-occupancy is still desirable even if it increases a (positive) fiscal externality. In *Table 3*, the last two rows show the impact on social welfare of increases in a marginal (*l* commodity) overlap of the tax base when the tax base is optimal divided for several alternative parameterizations.

Underlying suggestions to eliminate co-occupancy to eliminate fiscal externalities might be the implicit assumption that $x_{2l} = 0$. If this is the case, the only fiscal externalities arising occur due to the overlap in the tax base. If $x_{2l} \neq 0$, specifically if $x_{2l} > 0$, with all commodities being gross substitutes, elimination of a fiscal externality arising from taxes requires that the tax bases be set so that

$$\frac{\partial R_s}{\partial \tau_l} = \tau_s (k_{ls} x_{11} + (k_{ls} + k_s)(k_{ls} + k_l) x_{21}) = 0. \quad (4.11)$$

If $k_{ls} > 0$, the fiscal externality associated with an increase in the local tax base is given by

$$\frac{\partial R_s}{\partial k_l} = \tau_s (x_{11} + (k_{ls} + k_s) x_{21}). \quad (4.12)$$

If the tax bases are set so that the fiscal externality associated with changes in tax rates are eliminated, that is (4.11) is satisfied, using (4.11) in (4.12) gives

$$\left. \frac{\partial R_s}{\partial k_l} \right|_{\frac{\partial R_s}{\partial \tau_l} = 0} = \tau_s k_s (x_{11} + x_{21}) < 0. \quad (4.13)$$

While the fiscal externality associated with the tax rate is eliminated by the overlap in the tax base, the fiscal externality associated with the tax base will not be eliminated. With all commodities being gross substitutes, the overlap needed to eliminate fiscal externalities from the tax rate generates a negative fiscal externality associated with the tax base. In the absence of $x_{2l} = 0$, the fiscal externalities associated with tax rates are distinct from those associated with the bases and elimination of the fiscal externalities from taxes will not eliminate the fiscal externalities associated with the tax bases.

5. *Optimal Tax Bases with Intergovernmental Grants*

The result that co-occupancy is not optimal if the tax base is optimally divided means that in general an optimally designed tax base will not eliminate fiscal externalities. This, of course, suggests that there is still a role for intergovernmental (matching) grants that internalize any fiscal externalities associated with the tax rates. If intergovernmental grants are used, then, what should the optimal division of the tax base be? The second-best outcome, given the use of distorting taxes, would be to have equal tax rates on all commodities and to have $MRS_s = MRS_l$. Here, I demonstrate that this outcome can be obtained even if $x_{2l} \neq 0$ but only when both governments tax the entire tax base and the appropriate matching grant (tax) is imposed on local governments.

I assume throughout this discussion that the fiscal externality is only caused by the local government's tax policies, that is, $\alpha_s = 1$. Then following Dahlby (1996) and Hoyt (2001), for example, let there be a matching grant (tax) imposed on the local government such that the local government's budget constraint is given by:

$$(1 - m)\tau_l X_l = g_l \quad (5.1)$$

where m is the matching grant with $m > 0$ implying a transfer of funds from the local governments to the state government. The state budget constraint is given by

$$\tau_s X_s + m\tau_l X_l = g_s \quad (5.2)$$

In contrast to both Dahlby (1996) and Hoyt (2001), a single, uniform tax rate is applied on all commodities in a government's tax base. Specifically, the tax rate on commodities in the co-occupied tax base is the same as the rate on the parts of the tax bases that do not overlap. Consistent with this tax structure, a single matching rate is also set for any commodity in the local government's tax base.¹⁵

Then following Dahlby (1996) and Hoyt (2001) set the matching rate, m , so that the fiscal externality is eliminated, that is,

$$\frac{\partial g_s}{\partial \tau_l} = \tau_s \frac{\partial X_s}{\partial \tau_l} + m \left(X_l + \frac{\partial X_l}{\partial \tau_l} \right) = 0, \quad (5.3)$$

or

$$m = -\tau_s \frac{\partial X_s}{\partial \tau_l} \left(X_l + \frac{\partial X_l}{\partial \tau_l} \right)^{-1}. \quad (5.3')$$

With this matching rate, the first order condition for local governments can be expressed by

$$MRS_l = \frac{X_l}{X_l + \tau_l \frac{\partial X_l}{\partial \tau_l} + \tau_s \frac{\partial X_s}{\partial \tau_l}} \quad (5.4)$$

The first order condition for the state government, with $a_s = 1$, is

$$-X_s + MRS_s \frac{\partial g_s}{\partial \tau_s} + MRS_l \frac{\partial g_l}{\partial \tau_s} = 0. \quad (5.5)$$

If $MRS_l = MRS_s$ then (5.5) can be expressed as

$$MRS_s = MRS_l = \frac{X_s}{X_s + \tau_l \frac{\partial X_l}{\partial \tau_s} + \tau_s \frac{\partial X_s}{\partial \tau_s}}.^{16} \quad (5.6)$$

Then using the expressions for MRS_l from (5.4) and for MRS_s (and MRS_l) in (5.6) we can see that $MRS_s = MRS_l$ only if

$$\frac{X_l}{X_l + \tau_l \frac{\partial X_l}{\partial \tau_l} + \tau_s \frac{\partial X_s}{\partial \tau_l}} = \frac{X_s}{X_s + \tau_l \frac{\partial X_l}{\partial \tau_s} + \tau_s \frac{\partial X_s}{\partial \tau_s}} \quad (5.7)$$

¹⁵ Dahlby (1996) and Hoyt (2001) both allow matching grants that vary with the commodity. In the framework used here that would imply different rates for commodities in the co-occupied section of the tax base and for the commodities in the section of the tax base only taxed by the local government.

¹⁶ Expression (5.6) is found by setting $MRS_l = MRS_s$ and substituting $\frac{\partial g_s}{\partial \tau_s} = X_s + \frac{\partial X_s}{\partial \tau_s} + m\tau_l \frac{\partial X_l}{\partial \tau_s}$ and $\frac{\partial g_l}{\partial \tau_s} = (1-m)\tau_l \frac{\partial X_l}{\partial \tau_s}$.

Equation (5.7) will only be satisfied with $x_{2i} \neq 0$ if the two tax bases are the same, $X_i = X_j$.

Then the optimal tax bases for the two governments with and without the matching grant are profoundly different. If matching grants are not available or are not set to eliminate the fiscal externalities associated with the tax rates, there should be no overlap in the two level of governments tax bases; if the fiscal externalities from taxes are eliminated by some other policy intervention, such as a matching grant, complete co-occupancy of the tax base, the unconstrained choices of the governments, is welfare-maximizing.

6. *Extensions and Conclusion*

While most literature on vertical fiscal externalities is very recent, there are some suggested policies to correct for inefficiencies associated with the existence of these vertical fiscal externalities that have begun to emerge. The most frequent policy recommendation is the use of intergovernmental grants to correct any misallocation of funds between levels of government and to force governments to internalize the fiscal externality. Another suggested policy is to reduce the existence of fiscal externalities by limiting the co-occupancy of the tax base.

It is this suggested remedy that is the focus of this paper. In fact, I find that complete elimination of co-occupancy is optimal if the tax base is optimally divided in the absence of co-occupancy and other corrective policies are not available. This result is true regardless of the cross-price elasticities among commodities and whether or not the vertical fiscal externality is positive or negative. However, this policy generally does not lead to the governments setting social-welfare maximizing tax rates, thus suggesting that other corrective policies are still desirable. If other corrective policies are used to eliminate fiscal externalities, complete co-occupancy of the tax base, not the elimination of co-occupancy, is socially optimal to ensure equal taxation of commodities (the optimal tax policy in this model) and that both governments face the same marginal cost of funds.

One extension to consider is the question of which commodities should be included in each tax base when commodities are not identical and they do not have identical cross-price elasticities. This extension would bring the analysis in closer alignment with the framework for addressing the assignment problem underlying Musgrave (1983). In a framework with collection costs, Wilson (1989) finds that when adding a com-

modity to the set of taxable commodities, it should be the strongest available substitute with the existing set of taxable commodities. Does a similar result apply here or does substitution with the tax base of the other government need to also be considered?

Another possible extensions merit further research. In the simple model presented here there were no horizontal fiscal externalities as there was no flow of tax base between different localities. Recent studies by Wilson and Janeba (2005) and Flochel and Madies (2002) consider both vertical and horizontal fiscal externalities though not with multiple tax bases. Relatively simple adjustments to the model would provide the opportunity to consider how local and state tax bases should be designed when these externalities also exist. It would seem likely that commodities or tax bases that flow between localities ideally would not be included in local tax bases.

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Table 1: Share of Tax Revenue, State and Local Governments by Source by State, 2000

(Top row: State Sources; Bottom Row: Local Sources)

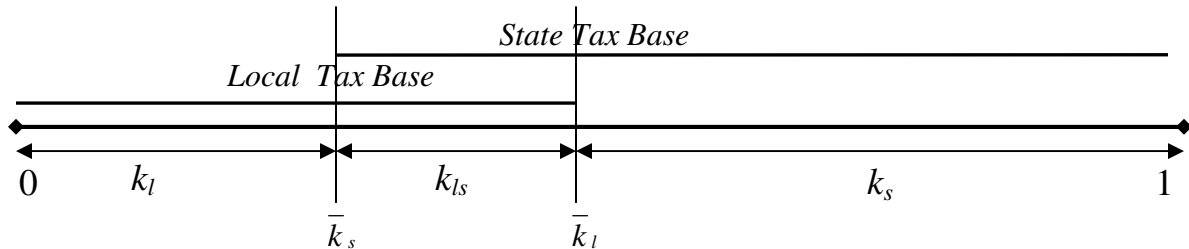
State	Property	Sales, All	Sales, General	Sales, Selected	Motor Fuel	Alcoholic Beverages	Tobacco	Income, Individual	Income, Corporate	Taxes, Other	Charges
U.S. Federal	0	8	0	8	2.4	0.7	0.6	65.3	17.1	5.6	---
All States	2.4	49.3	33.3	16.0	6.2	0.9	1.8	31.9	7.0	6.4	13.5
All Local	73.7	15.8	11.0	4.8	0.3	0.1	0.1	4.9	1.0	4.1	20.9
Alabama	2.8	50.1	26.4	23.7	7.8	1.9	1.0	32.2	3.8	8.1	17.2
	39.0	45.6	39.2	6.4	1.6	1.1	0.5	2.9	0.0	11.7	27.7
Alaska	3.1	9.7	0.0	9.7	3.0	0.9	3.4	0.0	30.8	53.9	54.8
	80.7	16.4	12.0	4.4	0.0	0.1	0.5	0.0	0.0	1.7	21.7
Arizona	3.7	57.3	44.8	12.5	7.3	0.6	2.0	28.3	6.5	2.4	12.5
	69.0	27.3	23.3	3.9	0.0	0.0	0.0	0.0	0.0	3.8	17.4
Arkansas	9.9	48.5	35.0	13.5	8.0	0.6	1.9	30.2	4.9	4.1	14.0
	44.4	53.6	45.2	8.4	0.0	0.1	0.0	0.0	0.0	2.1	22.5
California	4.0	35.3	28.0	7.4	3.6	0.3	1.5	47.2	7.9	3.4	9.2
	63.2	27.8	19.3	8.5	0.0	0.0	0.0	0.0	0.0	9.0	21.2
Colorado	0.0	38.3	26.1	12.2	7.7	0.4	1.0	51.4	4.7	3.1	14.9
	59.9	34.8	31.4	3.5	0.0	0.0	0.0	0.0	0.0	4.7	25.1
Connecticut	0.0	49.7	33.6	16.1	5.3	0.4	1.3	39.1	4.2	4.8	14.8
	98.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.3	10.0
Delaware	0.0	13.6	0.0	13.6	4.9	0.5	1.3	34.4	11.3	39.2	26.6
	78.6	1.7	0.0	1.7	0.0	0.0	0.0	8.5	0.0	11.2	19.8
Florida	3.1	77.1	60.5	16.6	6.5	2.3	1.8	0.0	4.8	11.3	12.7
	77.9	18.3	3.2	15.1	3.2	0.0	0.0	0.0	0.0	3.6	28.2
Georgia	0.4	42.5	34.3	8.3	4.7	1.0	0.6	47.1	5.3	3.0	11.6
	60.4	36.5	29.8	6.7	0.0	1.1	0.0	0.0	0.0	3.1	22.0
Idaho	0.0	61.5	46.1	15.5	2.2	1.2	1.3	31.9	2.3	2.0	18.2
	78.6	11.6	0.0	11.6	7.9	0.0	0.0	0.0	0.0	3.2	25.2
Illinois	0.0	44.4	31.4	12.9	8.8	0.3	1.2	40.6	5.3	5.2	13.5
	94.6	1.6	0.0	1.6	0.0	0.0	0.0	0.0	0.0	3.3	26.7
Indiana	0.2	47.7	28.1	19.6	6.0	0.6	2.1	33.5	9.9	4.2	11.9
	82.8	14.4	5.1	9.4	1.1	0.0	0.4	0.0	0.0	2.3	16.8
Iowa	0.0	49.8	35.4	14.4	6.9	0.3	0.9	37.1	9.2	2.2	18.6
	88.6	1.2	0.0	1.2	0.0	0.0	0.0	8.2	0.0	1.9	24.5
Kansas	0.0	47.8	33.2	14.6	6.7	0.2	1.9	36.5	4.1	5.1	17.4
	89.5	7.5	5.9	1.7	0.0	0.0	0.0	1.3	0.0	1.2	24.1
Kentucky	1.0	47.5	35.9	11.6	7.3	1.5	1.1	38.4	5.6	4.5	12.6
	76.8	21.0	17.0	4.0	0.0	0.0	0.0	0.0	0.0	2.1	22.8
Louisiana	5.1	45.6	28.2	17.4	5.7	0.9	0.2	35.1	4.0	7.7	13.4
	53.8	7.0	0.0	7.0	0.0	0.0	0.0	30.1	0.0	8.4	23.4
Maine	0.4	57.1	31.6	25.5	8.4	0.8	1.4	24.3	3.4	13.1	19.0
	39.3	57.3	51.7	5.6	0.0	0.1	0.0	0.0	0.0	3.3	23.1
Maryland	1.1	44.8	31.8	12.9	6.8	1.3	2.8	40.5	5.6	5.4	16.8
	97.9	0.3	0.0	0.3	0.0	0.0	0.0	0.0	0.0	1.0	15.3
Massachusetts	2.5	42.1	24.1	18.0	6.3	0.2	2.0	44.6	4.2	5.0	16.1
	57.4	3.3	0.0	3.3	0.0	0.0	0.0	32.0	0.0	7.3	16.2
Michigan	0.0	31.3	22.1	9.3	4.0	0.4	1.7	56.0	8.1	3.2	17.1
	96.9	1.2	0.0	1.2	0.0	0.0	0.0	0.0	0.0	1.9	10.9
Minnesota	7.5	43.0	33.7	9.3	4.7	0.6	2.7	31.6	10.5	3.9	14.9
	89.4	1.4	0.0	1.4	0.0	0.0	0.0	6.2	0.0	3.0	19.9
Mississippi	0.1	43.3	27.9	15.4	4.6	0.5	1.4	41.6	6.0	4.2	11.8
	94.2	2.8	0.7	2.1	0.0	0.0	0.0	0.0	0.0	2.9	25.1

<i>Table 1 (continued)</i>											
State	Property	Sales, All	Sales, General	Sales, Selected	Motor Fuel	Alcoholic Beverages	Tobacco	Income, Individual	Income, Corporate	Taxes, Other	Charges
Missouri	0.0	66.8	49.5	17.3	8.9	0.8	1.2	21.4	4.8	4.6	11.9
	92.0	3.5	0.0	3.5	0.5	0.0	0.0	0.0	0.0	4.5	31.2
Montana	0.2	47.0	32.5	14.5	8.1	0.3	1.3	41.4	3.1	5.4	13.1
	59.0	31.4	23.0	8.4	0.0	0.0	0.3	5.2	0.0	4.2	21.1
Nebraska	15.5	24.4	0.0	24.4	13.4	1.2	1.0	36.6	7.1	12.5	20.9
	95.6	0.2	0.0	0.2	0.0	0.0	0.0	0.0	0.0	2.4	25.3
Nevada	0.1	48.5	34.5	14.0	9.4	0.6	1.6	39.4	4.7	4.5	18.7
	77.5	12.3	9.5	2.8	0.0	0.0	0.0	0.0	0.0	9.2	17.2
New Hampshire	2.5	84.8	52.2	32.6	7.0	0.4	1.7	0.0	0.0	9.6	9.6
	63.8	20.5	5.7	14.8	3.0	0.0	0.0	0.0	0.0	15.7	25.8
New Jersey	27.9	32.8	0.0	32.8	6.9	0.7	5.6	3.9	18.4	13.2	20.5
	98.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.8	12.2
New Mexico	0.0	45.2	30.4	14.8	2.8	0.4	2.2	39.7	7.4	5.7	15.0
	98.3	0.2	0.0	0.2	0.0	0.0	0.0	0.2	0.0	1.2	15.2
New York	0.9	53.6	40.1	13.5	6.2	1.0	0.6	23.5	4.3	13.9	18.5
	55.4	40.3	34.6	5.8	0.0	0.0	0.0	0.0	0.0	4.3	18.1
North Carolina	0.0	31.9	20.5	11.4	1.2	0.4	1.6	55.6	6.6	4.4	9.4
	55.8	20.3	17.5	2.8	0.0	0.0	0.1	12.1	7.3	4.2	15.0
North Dakota	0.0	38.8	22.0	16.8	7.0	1.3	0.3	47.1	7.8	3.8	11.2
	75.2	20.6	18.7	1.9	0.0	0.1	0.0	0.0	0.0	3.9	25.9
Ohio	0.2	55.9	28.2	27.7	9.4	0.5	1.9	16.9	6.7	16.6	18.9
	88.1	9.9	8.6	1.3	0.0	0.0	0.0	0.0	0.0	2.0	22.2
Oklahoma	0.1	46.0	31.8	14.2	7.1	0.4	1.5	41.9	3.2	5.7	11.6
	65.4	8.9	8.0	0.9	0.1	0.1	0.0	22.0	0.0	3.0	19.9
Oregon	0.0	37.3	24.7	12.7	6.9	1.0	1.3	36.5	3.3	11.9	14.5
	54.0	43.7	39.9	3.8	0.0	0.0	0.0	0.0	0.0	2.2	27.8
Pennsylvania	0.0	12.2	0.0	12.2	8.0	0.2	3.2	68.9	6.8	5.8	16.5
	80.5	5.3	0.0	5.3	0.3	0.0	0.0	0.0	0.0	14.0	24.2
Rhode Island	0.5	46.6	31.4	15.1	3.4	0.8	1.4	30.1	7.6	11.7	15.9
	70.5	2.7	1.2	1.5	0.0	0.0	0.0	17.8	0.0	9.0	17.9
South Carolina	0.2	50.8	38.5	12.3	5.8	2.0	0.5	38.3	3.6	5.4	16.0
	84.4	7.6	3.1	4.4	0.0	0.0	0.0	0.0	0.0	7.6	29.7
South Dakota	0.0	79.0	52.6	26.4	13.5	1.2	2.1	0.0	4.9	11.9	19.4
	78.2	17.5	17.2	0.3	0.0	0.0	0.0	0.0	0.0	2.0	17.8
Tennessee	0.0	75.0	57.4	17.6	10.2	1.0	1.1	2.3	7.9	11.7	10.9
	61.5	32.2	26.7	5.5	0.0	2.0	0.0	0.0	0.0	4.2	21.3
Texas	0.0	81.0	51.1	29.9	9.8	1.9	1.9	0.0	0.0	15.6	14.8
	79.9	17.6	13.5	4.1	0.0	0.0	0.0	0.0	0.0	1.4	21.4
Utah	0.0	48.4	35.8	12.6	8.3	0.6	1.2	41.5	4.4	3.8	17.9
	68.8	26.7	22.1	4.6	0.0	0.0	0.0	0.0	0.0	4.5	20.1
Vermont	27.3	32.6	14.5	18.1	4.1	1.0	1.7	29.1	3.0	5.6	16.5
	96.2	0.3	0.0	0.3	0.0	0.0	0.0	0.0	0.0	3.5	11.1
Virginia	0.3	34.4	19.5	14.9	6.4	1.0	0.1	54.0	4.5	4.4	19.0
	70.6	19.2	8.8	10.4	0.0	0.0	0.4	0.0	0.0	8.8	17.7
Washington	13.5	77.1	61.6	15.5	6.2	1.3	2.2	0.0	0.0	7.1	11.4
	61.5	29.1	19.1	10.0	0.0	0.0	0.0	0.0	0.0	7.4	23.6
West Virginia	0.1	53.8	27.4	26.4	7.2	0.3	1.0	28.9	6.5	8.0	14.9
	83.6	3.6	0.0	3.6	0.0	0.3	0.0	0.0	0.0	12.8	24.4
Wisconsin	0.7	40.6	27.9	12.7	7.3	0.3	2.1	47.3	4.6	4.4	12.1
	93.8	3.8	3.2	0.6	0.0	0.0	0.0	0.0	0.0	2.4	16.2
Wyoming	10.5	49.5	38.3	11.2	8.4	0.1	1.1	0.0	0.0	34.6	9.4
	76.0	19.8	17.6	2.2	0.0	0.0	0.0	0.0	0.0	2.5	30.0

Table 2: Number of Overlapping Commodities at which EMCF = 0						
	Own-Price Elasticity of Demand					
	-1.25	-1.5	-2	-3	-4	-5
Local Tax Base	25	25	25	25	25	25
Overlapping Base	4.5	8.4	15.1	25.2	32.3	37.7
State Tax Base	70.5	66.6	59.9	49.8	42.7	37.3
Local Tax Base	50	50	50	50	50	50
Overlapping Base	5.6	10.1	16.8	25.1	30.1	33.4
State Tax Base	44.4	39.9	33.2	24.9	19.9	16.6
Local Tax Base	75	75	75	75	75	75
Overlapping Base	4.0	6.9	10.8	15.1	17.4	18.8
State Tax Base	21.0	18.1	14.2	9.9	7.6	6.2

Table 3: A Numerical Determination of the Optimal Division of the Tax Base												
	1 ($x_{11} = -1, x_{21} = 0$)				2 ($x_{11} = -.5, x_{21} = -.01$)				3 ($x_{11} = -2, x_{21} = 0.01$)			
	Ratio of Services (g_l/g_s)	1.00	1.67	3.00	7.00	1.00	1.67	3.00	7.00	1.00	1.67	3.00
Local Public Service (g_l)	10.02	12.52	15.03	17.53	10.83	13.41	15.80	16.11	8.76	11.27	13.97	16.89
State Public Service (g_s)	10.02	7.51	5.01	2.50	10.83	8.12	5.36	1.34	8.76	6.41	4.18	2.05
Obtained Ratio of Services	1.00	1.67	3.00	7.00	1.00	1.65	2.95	12.03	1.00	1.76	3.34	8.23
Local Tax Base (k_l)	50	62.5	75	87.5	50	65.0	78.6	92.4	50	63	75.8	88.2
Local MRS (MRS_l)	1.62	1.62	1.62	1.62	1.54	1.55	1.57	1.72	1.78	1.75	1.71	1.67
State MRS (MRS_s)	1.62	1.62	1.62	1.62	1.54	1.54	1.55	2.52	1.78	1.81	1.84	1.87
Ratio of MRS	1.00	1.00	1.00	1.00	1.00	1.01	1.01	0.68	1.00	0.96	0.93	0.89
Local Tax Rate (τ_l)	0.28	0.28	0.28	0.28	0.32	0.30	0.29	0.24	0.23	0.24	0.25	0.26
State Tax Rate (τ_s)	0.28	0.28	0.28	0.28	0.32	0.35	0.38	0.15	0.23	0.22	0.21	0.21
Ratio of Tax Rates	1.00	1.00	1.00	1.00	1.00	0.86	0.75	1.58	1.00	1.08	1.17	1.26
$W_{k_l} \big _{\bar{k}_l=k_l^p}$	-0.12	-0.12	-0.12	-0.12	-0.12	-0.11	-0.11	-0.35	-0.14	-0.15	-0.16	-0.17
$-W_{k_s} \big _{\bar{k}_l=k_l^p}$	-0.12	-0.12	-0.12	-0.12	-0.12	-0.13	-0.14	-0.18	-0.14	-0.13	-0.12	-0.12

Figure 1: Distribution of the Tax Base



Appendix

A.1 Impacts of Changes in Tax Bases and Rates on Revenue

Differentiating (2.6) gives the impact of an increase in the state tax ($j=s$) or in the local tax rate ($j=1$) on the tax bases,

$$\frac{\partial[\tau_j X_j]}{\partial \tau_j} = (k_{ls}x_{ls} + k_j x_j) + (k_{ls} + k_j)\tau_j(x_{11} + (k_{ls} + k_j)x_{21}), \quad j=l, s \quad (A.1.1a)$$

and

$$\frac{\partial[\tau_i X_i]}{\partial \tau_j} = \tau_u (k_{ls}x_{11} + (k_{ls} + k_i)(k_{ls} + k_j)x_{21}), \quad j, i=l, s; j \neq i. \quad (A.1.1b)$$

Then the impact of increasing the local tax base on local revenues is given by

$$\frac{d[\tau_l X_l]}{d\bar{k}_l} = \tau_l [x_z + \tau_s (k_{ls} + k_l)x_{21}], \quad z = ls \text{ if } \bar{k}_l \geq \bar{k}_s, \quad z = l, \text{ if } \bar{k}_l < \bar{k}_s \quad (A.1.2a)$$

and the impact on state revenue is given by

$$\frac{d[\tau_s X_s]}{d\bar{k}_l} = \tau_l \tau_s [Dx_{11} + (k_{ls} + k_s)x_{21}], \quad D = 1 \text{ if } \bar{k}_l \geq \bar{k}_s, \quad D = 0, \text{ if } \bar{k}_l < \bar{k}_s. \quad (A.1.2b)$$

Analogously, the impact of decreasing the state base (increasing \bar{k}_s) on state tax revenues is given by

$$\frac{d[\tau_s X_s]}{d\bar{k}_s} = -\tau_s [x_z + \tau_s (k_{ls} + k_s)x_{21}], \quad z = ls \text{ if } \bar{k}_l \geq \bar{k}_s, \quad z = s, \text{ if } \bar{k}_l < \bar{k}_s \quad (A.1.3a)$$

and the impact of decreasing the state tax base on local revenue is given by

$$\frac{d[\tau_l X_l]}{d\bar{k}_s} = -\tau_l \tau_s [Dx_{11} + (k_{ls} + k_l)x_{21}], \quad D = 1 \text{ if } \bar{k}_l \geq \bar{k}_s, \quad D = 0, \text{ if } \bar{k}_l < \bar{k}_s. \quad (A.1.3b)$$

where I approximate the change in demand for other commodities as a result of the decrease in price of $x(\bar{k}_j)$ using the first order approximation $dx(k) = \frac{\partial x(k)}{\partial q(\bar{k}_j)}(-\tau_j)$. The difference in the demand for commodity $x(\bar{k}_j)$ with and without the tax by government j is approximated by $-x_{11}\tau_j$. Recall that increasing \bar{k}_s is decreasing the state tax base and hence the opposite signs of (2.8) for an increase in \bar{k}_s and an increase in \bar{k}_l .

A.2 Derivations and Proofs from Section 3

A.2.1 The First Order Conditions for Choice of Tax Base and Tax Rate

Derivation of (3.2a):

Differentiating (3.1) with respect to τ_j :

$$\frac{\partial W^j}{\partial \tau_j} = \int_0^{\bar{k}_l} \frac{\partial V(q_j(k))dk}{\partial q(i)} + \frac{\partial V^l}{\partial g_j} \frac{\partial[\tau_j X_j]}{\partial \tau_j} + \alpha_l \frac{\partial V^s}{\partial g_s} \frac{\partial[\tau_s X_s]}{\partial \tau_j}, \quad j=1, \dots, n \quad (A.2.1)$$

From the first order conditions for consumers we have:

$$\int_0^{\bar{k}_s} U'(x(i))di = k_l U'(x_l) = \lambda k_l (1 + \tau_l) x_l; \int_0^{\bar{k}_l} U'(x(i))di = k_{ls} U'(x_l) = \lambda k_{ls} (1 + \tau_l) x_l; \quad (A.2.2)$$

$$\int_{k_l}^1 U'(x(i))di = k_s U'(x_l) = \lambda k_s (1 + \tau_l) x_l$$

For simplicity assume that $k_{ls} = 0$. Then we have

$$\int_0^1 V(q(i))di = \int_0^{\bar{k}_l} U(x(q(i) = 1 + \tau_l))di + \int_{k_l}^1 U(x(q(i) = 1 + \tau_s))di \quad (A.2.3)$$

Then differentiating with respect to τ_l gives:

$$\begin{aligned} \frac{\partial \int_0^1 V(q(i))di}{\partial \tau_l} &= \int_0^{\bar{k}_l} U'(x(q(i) = 1 + \tau_l)) \left(\frac{\partial x(i)}{\partial q(i)} + \int_0^{\bar{k}_l} \frac{\partial x(i)}{\partial q(j)} dj \right) di + \int_{k_l}^1 U'(x(q(i) = 1 + \tau_l)) \left(\int_0^{\bar{k}_l} \frac{\partial x(i)}{\partial q(j)} dj \right) di \\ &= \int_0^{\bar{k}_l} U'(x(q(i) = 1 + \tau_l)) x_{ll} di + \int_{k_l}^1 U'(x(q(i) = 1 + \tau_l)) x_{sl} di \\ &\text{where } x_{ll} = \frac{\partial x(i)}{\partial q(i)} + \int_0^{\bar{k}_l} \frac{\partial x(i)}{\partial q(j)} dj \text{ and } x_{sl} = \int_0^{\bar{k}_l} \frac{\partial x(i)}{\partial q(j)} dj \end{aligned} \quad (A.2.4)$$

Then from the first order conditions we have:

$$\frac{\partial W}{\partial \tau_l} = \lambda [k_l (1 + \tau_l) x_{ll} + k_s (1 + \tau_s) x_{sl}] \quad (A.2.5)$$

Let the budget constraint be expressed as:

$$\int_0^{\bar{k}_l} (1 + \tau_l) x(q(i) = 1 + \tau_l) di + \int_{k_l}^1 (1 + \tau_s) x(q(i) = 1 + \tau_s) di = y \quad (A.2.6)$$

Then differentiating the budget constraint gives:

$$\begin{aligned} \int_0^{\bar{k}_l} (1 + \tau_l) \left(\frac{\partial x(i)}{\partial q(i)} + \int_0^{\bar{k}_l} \frac{\partial x(i)}{\partial q(j)} dj \right) di + \int_{k_l}^1 (1 + \tau_s) \left(\int_0^{\bar{k}_l} \frac{\partial x(i)}{\partial q(j)} dj \right) di + \int_0^{\bar{k}_l} x(i, q(i) = 1 + \tau_l) di &= 0 \\ &= \int_0^{\bar{k}_l} (1 + \tau_l) x_{ll} di + \int_{k_l}^1 (1 + \tau_s) x_{sl} di + k_l x_l = 0 \end{aligned} \quad (A.2.7)$$

Then using (A.2.7) in (A.2.5) gives

$$\frac{\partial W}{\partial \tau_l} = -\lambda x_l \quad (A.2.8)$$

Then using (A.2.8) (with $\lambda = V_y$) and (A.1.2a and A.1.2b) in (A.2.1) gives (3.2a).

Derivation of (3.2b):

Differentiating the welfare function for locality j with respect to its tax base, \bar{k}_l , gives:

$$\frac{\partial W^j}{\partial \bar{k}_l} = \frac{d \int_0^1 V(q(i))di}{dq(\bar{k}_l)} + \frac{\partial V^l}{\partial g_j} \frac{\partial [\tau_j X_j]}{\partial \bar{k}_l} = 0, \quad j = 1, \dots, n. \quad (A.2.9)$$

Then given the discrete change in the price of $x(\bar{k}_l)$ I use the first order approximation of $\int_0^{\bar{k}_l} \frac{\partial V(q(k))}{\partial q(\bar{k}_l)} d\tau_k =$

$-V_y(x(\bar{k}_l, q(\bar{k}_l) = 1 + \tau_l)) \tau_l$. Using (A.1.3a) and (A.1.3b) in (A.2.9) gives (3.2b).

A.2.2 Determination of the Choice of Tax Base

Evaluate (3.2b) at $k_{ls} = 0$ this gives

$$W_{k_l}^l = V_y \tau_l [-x_l + MRS_l(x_l + \tau_l k_l x_{21})] \quad (A.2.10)$$

The first order condition for the tax rate when $k_{ls}=0$ (from (3.2a)) is

$$W_{\tau_l}^l = V_y k_l [(MRS_l - 1)x_l + MRS_l \tau_l (x_{11} + k_l x_{21})] = 0 \quad (A.2.11)$$

Then subtracting (A.2.11) from (A.2.10) gives (3.8a).

Evaluating (3.2b) with $k_{ls} > 0$ gives

$$W_{k_l}^l = V_y \tau_l [-x_{ls} + MRS_l(x_{ls} + \tau_l(k_l + k_{ls})x_{21})] \quad (A.2.12)$$

and evaluating (3.2a) with $k_{ls} > 0$ gives

$$W_{\tau_l}^l = V_y [(MRS_l - 1)(k_l x_l + k_{ls} x_{ls}) + MRS_l \tau_l (k_{ls} + k_s)(x_{11} + (k_l + k_{ls})x_{21})] = 0 \quad (A.2.13)$$

Then using (A.2.13) in (A.2.12) gives

$$W_{\tau_l}^l = \frac{V_y \tau_l}{(k_l + k_{ls})} [(MRS_l - 1)k_l(x_{ls} - x_l) - (k_{ls} + k_s)MRS_l \tau_l x_{11}] = 0 \quad (A.2.14)$$

which, using $\tau_s x_{11} \approx x_{ls} - x_l$ can be simplified to obtain (3.8b).

A.2.3 Proof Proposition 2c)

From (3.2a) when $\alpha_s = 1$ and $\bar{k}_l = 1$

$$W_{\tau_l \bar{k}_l=1}^l = V_y [-x_{ls} + MRS_l(x_{ls} + \tau_l(x_{11} + x_{21}))] = 0 \quad (A.2.15)$$

From (3.3a) when $\bar{k}_s = 0$ and $\alpha_s = 1$,

$$W_{\tau_l \bar{k}_s=0}^l = V_y [-x_{ls} + MRS_s(x_{ls} + \tau_s(x_{11} + x_{21})) + MRS_l \tau_l(x_{11} + x_{21})] = 0 \quad (A.2.16)$$

Then subtracting (A.2.15) from (A.2.16) and solving for MRS_s gives

$$MRS_s = \frac{x_{ls}}{[x_{ls} + \tau_s(x_{11} + x_{21})]} MRS_l \quad (A.2.17)$$

A.2.3 Proposition 3 and Impacts of an Expansion of the Tax Base on Social Welfare

No co-occupancy ($k_{ls} = 0$):

In absence of any co-occupancy, ($k_{ls} = 0$), the impact of an expansion in a local tax base on the ($n-1$) other jurisdictions is:

$$\frac{\partial \sum_{\substack{i=1 \\ i \neq j}}^n V^i}{\partial k_j} = (n-1) V_{g_s} \tau_s \frac{\partial X_s}{\partial k_l} = (n-1) \frac{1}{n} \tau_s k_s x_{21} = \left(1 - \frac{1}{n}\right) V_{g_s} \tau_s k_s x_{21} \quad (A.2.18)$$

using (A.1.3). The total impact on welfare is the sum of (3.8a) and (A.2.18) and $\frac{1}{n} V_{g_s} \tau_s \frac{\partial X_s}{\partial k_l}$, the impact of the change in state services on the residents of locality j . Then we obtain (3.10a).

Complete Co-occupancy ($\bar{k}_l = 1, \bar{k}_s = 0$): With co-occupancy the impact of the other $n-1$ localities is

$$\frac{\partial \sum_{\substack{i=1 \\ i \neq j}}^n V^i}{\partial k_j} = V_y \tau_l \left[\frac{-x_{11}}{(k_l + k_{ls})} [MRS_l(k_l(\tau_l - \tau_s) + k_{ls} \tau_l) + k_{ls} \tau_s + MRS_s \tau_s(x_{11} + (k_s + k_{ls})x_{21})] \right] \quad (A.2.19)$$

Then as with the case when $k_b = 0$, the total impact on welfare is the sum of (3.8a) and (4.2.19) and

$\frac{1}{n} V_{\tau_s} \frac{\partial X_s}{\partial k_l}$, giving (3.11a). The impact of an increase in the state tax base, (3.11), can be found analogously.

Proposition 3b)

Note that

$$\left. \frac{\partial \sum_{\substack{i=1 \\ i \neq j}}^n V^i}{\partial k_l} \right|_{k_b=1} = V_y \tau_l \left[-x_{11} \left[(MRS_l \tau_l - MRS_s \tau_s) \right] + MRS_s \tau_s x_{21} \right] \quad (A.2.20)$$

and

$$\left. \frac{\partial \sum_{\substack{i=1 \\ i \neq j}}^n V^i}{\partial k_s} \right|_{k_b > 0} = -\tau_s V_y \left[-x_{11} \left[MRS_s \tau_s - (1 - \alpha_s) MRS_l \tau_l \right] + (1 - \alpha_s) MRS_l \tau_l x_{21} \right] \quad (A.2.21)$$

The first order conditions for taxes when $k_b = 1$:

$$W_{\tau_l}^l = V_y \left[(MRS_l - 1)x + MRS_l \tau_l (x_{11} + x_{21}) \right] = 0 \quad (A.2.22a)$$

$$W_{\tau_s}^s = V_y \left[-x + \alpha_s MRS_l \tau_l (x_{11} + x_{21}) + MRS_s (x + \tau_s (x_{11} + x_{21})) \right] = 0 \quad (A.2.22b)$$

Then from (A.2.22a):

$$MRS_l = \frac{x}{D_l}, \quad D_l = x + \tau_l S, \quad S = x_{11} + x_{21} \quad (A.2.23a)$$

and from (A.2.22b):

$$MRS_s = \frac{x - \alpha_s MRS_l \tau_l S}{D_s}, \quad D_s = x + \tau_s (x_{11} + x_{21}) \quad (A.2.23b)$$

and solving (A.2.23) gives:

$$MRS_l = \frac{x(x + \tau_s S)}{D}, \quad (A.2.24a)$$

$$MRS_s = \frac{x(x + (1 - \alpha_s) \tau_l S)}{D} \quad (A.2.24b)$$

where $D = D_l D_s$. With $\alpha_s = 1$ we have

$$MRS_l = \frac{(x^2 + \tau_s x S)}{D} \text{ and } MRS_s = \frac{x^2}{D} \rightarrow MRS_l < MRS_s \quad (A.2.25)$$

With $\alpha_s = 0$ we have

$$MRS_l = \frac{x(x + \tau_s S)}{D} \text{ and } MRS_s = \frac{x(x + \tau_l S)}{D} \quad (A.2.26)$$

Then

$$MRS_l - MRS_s = \frac{(\tau_s - \tau_l) S}{D} \quad (A.2.27)$$

and

$$MRS_l \tau_l - MRS_s \tau_s = \frac{\tau_l x(x + \tau_s S)}{D} - \frac{\tau_s x(x + \tau_s S)}{D} = \frac{(\tau_l - \tau_s) x^2}{D} \quad (A.2.28)$$

Evaluating the welfare impacts of an increase in the local tax base when $k_l = 1$ is can be found by using (A.2.27) in (A.2.20) to give (3.12a).

A.3 Derivations and Proofs from Section 4

A.3.1 Proof of Proposition 4

Part a): The first order conditions for taxes (with equal tax rates) as

$$MRS_l = \frac{x}{x + \tau(x_{11} + k_l x_{21})} \quad (A.4.1a)$$

$$MRS_s = \frac{x}{x + \tau(x_{11} + k_s x_{21})} - \alpha_s \frac{MRS_l \tau k_l x_{21}}{x + \tau(x_{11} + k_s x_{21})}. \quad (A.4.1b)$$

Then from (A.4.1) we solve for

$$MRS_l = \frac{x D_s}{D} = \frac{x}{D} [x + \tau(x_{11} + k_s x_{21})] \quad (A.4.2a)$$

and

$$MRS_s = \frac{x}{D} [x + \tau(x_{11} + k_l (1 - \alpha_s) x_{21})]. \quad (A.4.2b)$$

So for $MRS_s = MRS_l$ and $\tau_l = \tau_s$ it follows from (A.4.2) that

$$1 - k_l = k_l (1 - \alpha_s) \quad (A.4.3)$$

$$\text{giving } k_l = \frac{1}{2 - \alpha_s} \quad (A.4.4)$$

Part b) can be seen by using the fact that when $\alpha_s = 1$, $\tau_l = \tau_s = \tau$ we can express the first order conditions for taxes by

$$MRS_l = \frac{x}{x + \tau(x_{11} + k_l x_{21})} \quad \text{and} \quad (A.4.5a)$$

$$MRS_s = \frac{x}{x + \tau(x_{11} + k_s x_{21})} - MRS_l \tau_l k_l x_{21} \quad (A.4.5b)$$

Then setting $MRS_l = MRS_s$ in (A.4.5) gives

$$MRS_l = \frac{x}{x + \tau(x_{11} + k_l x_{21})} = \frac{x}{x + \tau(x_{11} + k_s x_{21})} - \tau_l k_l x_{21} \frac{x}{[x + \tau(x_{11} + k_l x_{21})][x + \tau(x_{11} + k_s x_{21})]} = MRS_s \quad (A.4.6)$$

Then simplifying we obtain:

$$MRS_l = x + \tau(x_{11} + k_s x_{21}) = x + \tau(x_{11} + k_l x_{21}) - \tau_l k_l x_{21} \rightarrow \tau k_s x_{21} = 0 \quad (A.4.6')$$

A.3.2 Proposition 5

Part a.i) and Part b.i) and b.ii), the conditions under which $\bar{k}_l^p > (<).5$ follows from the assumption that $W(\bar{k}_l, \tau_l(\bar{k}_l), \tau_s(\bar{k}_l))$ is strictly concave in \bar{k}_l .

a.ii) (Relative Tax Rates with $\alpha_s = 0$)

$$MRS_l = \frac{x_l}{x_l + \tau_l(x_{11} + k_l x_{21})} = \frac{x_l}{D_l} \quad \text{and} \quad MRS_s = \frac{x_s}{x_s + \tau_s(x_{11} + k_s x_{21})} = \frac{x_s}{D_s} \quad (A.4.7)$$

where $S_j = -(x_{11} + k_j x_{21})$. Then we can express the left side of (4.2) as

$$\frac{\tau_l^2 S_l x_l}{D_l} + \frac{\tau_l^2 k_l x_l x_{21}}{D_l} - \frac{\tau_l \tau_s k_l x_l x_{21}}{D_l} - \frac{\tau_s^2 S_s x_s}{D_s} + \frac{\tau_l \tau_s k_s x_s x_{21}}{D_l} - \frac{\tau_s^2 k_s x_s x_{21}}{D_l} \quad (A.4.8)$$

Simplifying using the fact that $S_j = -(x_{1l} + k_j x_{2l})$ gives

$$\left[\frac{-\tau_l^2 x_l}{D_l} + \frac{\tau_s^2 x_s}{D_s} \right]_{(a)} x_{11} + \tau_l \tau_s \left[\frac{-x_l k_l}{D_l} + \frac{x_s k_s}{D_s} \right]_{(b)} x_{21} = 0 \quad (A.4.8')$$

If $x_{21} > 0$, then from (A.4.8') we have $k_l^p > (<) k_s^p \rightarrow \tau_l^p > (<) \tau_s^p$ with the reverse true if $x_{21} < 0$.

a.iii) If we express (4.2) as

$$MRS_s \tau_s \left[-x_s + k_s x_{21} (\tau_l - \tau_s) \right]_{(a)} + MRS_l \tau_l \left[x_l + k_l x_{21} (\tau_l - \tau_s) \right]_{(b)} + (\tau_l x_l - \tau_s x_s)_{(c)} = 0. \quad (A.4.9)$$

Then if $x_{21} > 0$, then from *Proposition 5.b* we have $k_l^p > (<) k_s^p \rightarrow \tau_l^p > (<) \tau_s^p$. Then it follows that term (c) of (4.6) > 0 implying (a) + (b) < 0 . Since term (b) > 0 then term (a) < 0 . Further, the bracketed term in (a) is of smaller absolute value than the bracketed term in (b) ($k_l^* > k_s^*$) meaning that for the sum of (a) and (b) to be negative that $MRS_s \tau_s > MRS_l \tau_l$. Then since $\tau_l > \tau_s$ it must be the case that $MRS_s > MRS_l$. An analogous (and reversed) argument applies for the case with $x_{21} < 0$ and that $\tau_l(.5) > \tau_s(.5)$.