

**IMPERFECT COMPETITION, PRINCIPLES OF COMMODITY
TAXATION AND TAX INEFFICIENCIES IN FEDERAL SYSTEMS**

by

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First version: September 4, 2006

PRELIMINARY—COMMENTS WELCOME

Paper to be presented at the IFIR/CESifo Conference on ‘New Direction in Fiscal Federalism’, Lexington, Kentucky, September 14th-16th, 2006.

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1 Introduction

Recent work has emphasized that the commonality of tax base between the central and lower-level governments creates not only horizontal externalities (that arise when each lower-level government ignores the benefit a rise in its tax rate has on the revenues of the others) but also vertical externalities (that arise when lower-level governments unduly discount the reduction in federal revenues caused by a rise in their tax).¹ These externalities are moving towards opposite directions: Horizontal towards lower-level government taxes that are too low, from an efficient point of view, whereas vertical towards taxes that are, in the same sense, too high. It is so the case that the direction of the inefficiency in the lower-level government tax rates critically depends on the balance of these two externalities, Keen and Kotsogiannis (2002).

The conclusion regarding the sign of the vertical externality (and so the balance between this and the horizontal externality) is, typically, reached within models where the (mobile) factor of production is either capital or labor, there is perfect competition in the goods market, and the instruments available to the governments are specific taxes levied on the return to the mobile factor of production (and so to the demand of this factor). Notable exception is the contribution by Dahlby and Wilson (2003) who show, that the vertical fiscal externality may be either positive or negative if taxes levied are ad-valorem and not specific and that the sign of the vertical externality depends on which side of the market the tax is levied: If an ad-valorem tax is levied on employer payroll the vertical externality is always negative, whereas if it is levied on employees' wages it may generate a positive vertical fiscal externality. This latter possibility arises if the demand of labor is upward sloping and inelastic, so an increase in the tax rate, though it reduces employment, it does not reduce the wage tax base.

Though instructive and insightful, the contribution of Dahlby and Wilson (2003) is developed within a framework in which there is a perfectly competitive goods market, that takes prices perimetrically, and the factor of production is immobile. The implication of the latter assumption is that the issue of horizontal externalities cannot arise and so the issue regarding the direction of tax inefficiencies at the lower-level of government, once one accounts for both externalities, cannot be addressed. In addition to this a related issue that naturally arises is the extent to which the results that have appeared so far in the literature—developed within models of capital or labor tax competition—are robust to a setup where firms are imperfectly competitive² jurisdictions compete over commod-

¹There is now a fairly sizeable literature on vertical externalities. See, among others, Dahlby (1996), Wrede (1997), Boadway, Marchand and Vigneault (1998), Dahlby and Wilson, and Keen and Kotsogiannis (2002, 2003).

²And, therefore, firms follow a pricing policy that departs from marginal cost.

ity taxes, and taxes can be levied according to the destination or the origin principle.³ This latter possibility is briefly addressed by Dahlby and Wilson (2003) where they note that ‘...if the federal government levies an ad valorem tax and collects it from producers, then the market price includes the tax and the vertical fiscal externality from a state excise tax on the same commodity maybe positive if the demand for the taxed commodity is price inelastic. If the federal government levies an ad valorem tax and collects it from the purchaser of the commodity, then the market price does not include the tax and the vertical tax externality will be negative’, p. 923.

As it will be shown, in an imperfectly competitive environment things are more complicated. The focus of the analysis on imperfectly competitive markets make sense as in most markets firms are neither so small as to effectively take the market price as given (as in perfectly competitive markets) nor are there many cases of private-sector firms without any competition (as in the oligopolistic markets). The predominant market form is oligopoly. In these markets, typically, there are horizontal and vertical externalities: Horizontal, since jurisdictions compete over taxes, and vertical since these taxes are used extensively by federal governments too. In the U.S, for example, the gasoline (unit) tax accounts for over a third of total state sales tax revenues and over half of federal revenues. Other (unit) taxes are on motor fuel, tobacco products and alcoholic beverages. Examples of ad valorem taxes are taxes imposed on public utilities.

In this paper we ask:

- What is the direction (and magnitude) of fiscal externalities (horizontal and vertical) in a federal system where commodities are produced by an imperfectly competitive sector, are taxed under two different tax principles (destination or origin) and taxes can be specific or ad valorem? and
- what is the implication of these externalities for the level of lower-level government taxation?

The investigation of these issues is also particularly important for the EU. As part of its objective to create an efficient common market, the European Union has undertaken a legal and political commitment to establish a common system of commodity taxation based on the origin principle (commodities are taxed by—and revenues accrue to—the country that produces them). The achievement of this objective entails the shift from the current destination principle (commodities are taxed by—and revenues accrue to—the country where consumption takes place) to the origin principle. At the same time there are discussions at a EU level for the introduction of modulated VAT, advocated by

³The seminal contribution dealing with commodity tax competition is that of Mintz and Tulken (1986). For a brief survey see Lockwood (2001).

the European Parliament (1994). The Langes Report (1994), for example, has suggested using a harmonised VAT base. Both national parliaments and the EU would be granted the power to determine *separately* which rate would be imposed for purposes of the national budget and which for the EU budget, respectively. It is thus clear that the introduction of such tax responsibility at the EU level will give rise to vertical, that work along side any existing horizontal, fiscal inefficiencies. It is so of vital importance the investigation of the interplay between the two tax principles of taxation, destination and origin, and fiscal inefficiencies created by federal commodity tax (specific or ad valorem) competition.

Although there is some work dealing with the first question raised in the preceding paragraph (Keen (1998), Rizzo (2005), and Devereux, Lockwood and Redoano (2004)) there is no work, to the best of our knowledge, that makes an explicit comparison of tax inefficiencies between *alternative* commodity-tax principles. Keen (1998), focusing on commodity taxes, studies only vertical tax competition and so assumes that lower-level governments do not interact with each other. Rizzo (2003), and Devereux, Lockwood and Redoano (2004) emphasize (and empirically estimate the direction of) both types of inefficiencies but they do so in a commodity taxation framework with cross-border shoppers where there is no explicit production of the tradeable good, there are no public goods provided by both levels of government, lower-level governments are symmetric (in terms of size), and taxation is based on the destination principle (taxes are set and collected by the country where the good is consumed). Dahlby and Wilson (2003), as already noted, have looked at the direction of the vertical fiscal externality. But their goods market is one of perfect competition and they pay no attention to the balance of horizontal and vertical externalities for the level of lower-level government taxation.

This paper, in a standard model of imperfect markets, shows that:

- Under specific taxation, the direction of inefficiencies when both levels of government are present is not affected by the tax principle in place but it does *critically* depend on the deviation of the marginal valuation for the federal public good from the marginal valuation for the state public good. In particular, in the equilibrium considered, state taxes are too low (high), from an equilibrium point of view, under both destination and origin-based taxes, if consumer value the state more (less) than the federal public good.
- Under ad valorem taxation, things are more complicated. This is perhaps not surprising given the (non-additive) dependence of the market on prices. Nevertheless the analytics show that with ad valorem taxation the externalities in a federal system, though easy to be identified, they are cumbersome to be evaluated. For such an evaluation a precise knowledge of the fundamental parameters of the demand

and supply functions are required as well as the preferences of the consumers for the public goods.

- It is also shown, in passing, that under specific taxes the direction of inefficiencies of horizontal externalities differ between origin and destination tax regimes.

The paper is structured as follows. Section 2 provides the background against which the analysis is conducted. Section 3 deals with the fiscal externalities arising from specific destination-based commodity tax competition, while Section 4 deals with the fiscal externalities arising from specific origin-based federal tax competition. Section 5 analyses the case of ad valorem destination-based commodity tax competition, while Section 6 considers ad valorem origin-based commodity federal tax competition. Section 7 summarizes and concludes.

2 The background of the model

The model is one of oligopolistic competition, familiar enough from Kotsogiannis and Lopez-Garcia (*forthcoming*), appropriately modified to deal with issues of federal indirect tax competition. We consider a federal economy with two jurisdictions conveniently called ‘states’ labelled i and j . In each state there is an imperfectly competitive firm which produces a tradeable good. There is a representative consumer residing in each state, but demand between states may vary. The consumer in state i (j) may purchase this tradeable good by the firm located in either state. In both states there is also another good, which is taken as a numeraire, that is produced by perfectly competitive firms.⁴

Denoting Q^i (Q^j) the consumer price for the good produced in the imperfectly competitive sector in state i (j) the demand for the tradeable good is given by

$$D^i = a^i - \beta^i Q^i \quad ; \quad D^j = a^j - \beta^j Q^j \quad , \quad (1)$$

where a^i , β^i (a^j , β^j) are strictly positive demand parameters for state i (j).

Both firms have linear cost structures given by

$$C^i = c^i X^i + F^i \quad ; \quad C^j = c^j X^j + F^j \quad , \quad (2)$$

where X^i and X^j are the quantities produced, c^i and c^j are the (strictly positive) marginal costs of production and F^i and F^j are the (strictly positive) fixed costs of the firm located in state i and j , respectively. The tradeable good can be supplied from state i or state

⁴The production of both goods requires a single factor of production that is fixed in supply and so it is suppressed from the analysis.

j and, thus, either state can be an importer or exporter of the tradeable good. Market clearing for the federation, however, requires that

$$D^i + D^j = X^i + X^j . \quad (3)$$

We turn now to the description of the two types of taxes, specific and ad valorem, and the two tax principles, destination and origin. We start with the destination principle and specific taxation.

3 Destination principle and specific taxation

Recall that under the destination principle of taxation taxes are set, and collected, by the government of the state where the good is consumed. It is, thus, the case that arbitrage requires that producer prices, denoted by P^i for state i and P^j for state j , in both states will be equalized that is, $P^i = P^j = P$.

In the federation there is, as is typically the case, consolidating taxation in the sense that both levels of government tax the non-tradeable commodity. Denoting t_d^i and t_d^j the specific tax rate on consumption in state i and j respectively, and T_d the specific tax rate on consumption levied by the federal government to both states⁵ consumer prices in states i and j are then given, respectively, by

$$Q^i = P + t_d^i + T_d \quad ; \quad Q^j = P + t_d^j + T_d . \quad (4)$$

The revenues accrued to each state government are used to provide a non-tradeable public good, the price of which, in terms of the numeraire good, is fixed and for simplicity normalized to unity. The state government i 's (j) public good, denoted by g^i (g^j) is then given by

$$g^i = t_d^i D^i \quad ; \quad g^j = t_d^j D^j . \quad (5)$$

The federal government provides each state with tax receipts, denoted by G , given by⁶

$$G = (1/2)T_d(D^i + D^j) . \quad (6)$$

Notice that it is assumed throughout that there are no vertical intergovernmental transfers.⁷

⁵It is assumed, as is typically the case, that the federal government does not discriminate in the setting of taxes between the two states. This additive feature appears in many federation and, as noted in the introductory Section, it also appears to be behind the discussion of the Langes Report (1994) for the harmonised VAT base.

⁶This public good provision rule is not uncommon in the literature. See, for instance, Keen and Kotsogiannis (2002). One, of course, can define federal public good provision to be $G = T_d(D^i + D^j)$. This, however, does have some qualitative implication but the vertical externalities still remain.

⁷Vertical intergovernmental transfers are hard to rationalize when both levels have Nash conjectures. For such transfers, though in a different context, see Boadway and Keen (1996), and Kotsogiannis and Martinez (2006).

Substituting (4) into (1), and this into (3), gives the aggregate inverse demand function

$$P = b[a^i + a^j - \beta^i(t_d^i + T_d) - \beta^j(t_d^j + T_d) - (X^i + X^j)] , \quad \text{where } b \equiv 1/(\beta^i + \beta^j) > 0 . \quad (7)$$

The profits of state i 's (j 's) monopolistic firm are given by

$$\Pi^i = (P - c^i)X^i - F^i \quad ; \quad \Pi^j = (P - c^j)X^j - F^j , \quad (8)$$

with necessary condition, following from (7), given by

$$P - bX^i = c^i \quad ; \quad P - bX^j = c^j . \quad (9)$$

Equilibrium profits and so income for the representative consumer, making use of equation (9) in (8), are then given by

$$\Pi^i = b(X^i)^2 - F^i \quad ; \quad ; \quad \Pi^j = b(X^j)^2 - F^j . \quad (10)$$

Notice, for later use, that solving simultaneously equations (7) and (9) one obtains the inverse demand functions and the supply function for the tradeable good in states i and j given, respectively, by

$$P = (1/3)[b(a^i + a^j) - b\beta^i(t_d^i + T_d) - b\beta^j(t_d^j + T_d) + c_i + c_j] , \quad (11)$$

$$X^i = (1/3b)[b(a^i + a^j) - b\beta^i(t_d^i + T_d) - b\beta^j(t_d^j + T_d) - 2c^i + c^j] , \quad (12)$$

$$X^j = (1/3b)[b(a^i + a^j) - b\beta^i(t_d^i + T_d) - b\beta^j(t_d^j + T_d) - 2c^j + c^i] . \quad (13)$$

It will be proved particularly convenient for the identification of the types of externalities that are present, but also for intuition, to express the analytics throughout in terms of differentials of the actual variables. From (11)-(13) it, then, follows that

$$dP = -(b/3) [\beta^i (dt_d^i + dT_d) + \beta^j (dt_d^j + dT_d)] , \quad (14)$$

$$dX^i = dX^j = -(1/3) [\beta^i (dt_d^i + dT_d) + \beta^j (dt_d^j + dT_d)] . \quad (15)$$

Indirect utility in state i is of separable and additive, form and is given by

$$v^i(Q^i, \Pi^i, g^i, G) = u^i(D^i(Q^i)) - Q^i D^i(Q^i) + \Pi^i + \Gamma^i(g^i, G) , \quad (16)$$

where the first two terms correspond to the consumer surplus (the utility obtained from purchasing the tradeable good at price Q^i), the third term corresponds to the utility derived from income Π^i , whereas the final term $\Gamma^i(\cdot)$ gives the utility derived from public goods consumption in the state of residence of the consumer. An analogous expression holds for state j .

Total differentiation of equation (16), making use of (5) and (6), yields⁸

$$\begin{aligned} dv^i &= (X^i - D^i) dP + (P - c^i) dX^i + (\Gamma_g^i - 1) D^i dt_d^i + \Gamma_g^i t_d^i dD^i \\ &+ [((1/2)\Gamma_G^i - 1) D^i + (1/2)\Gamma_G^i D^j] dT_d + (1/2)\Gamma_G^i T_d dD^i + (1/2)\Gamma_G^i T_d dD^j . \end{aligned} \quad (17)$$

⁸The subscript $k = g, G$ denotes the partial derivative of $\Gamma(g, G)$ with respect to $k = g, G$.

Equation (17) simply says that utility in state i is affected by seven terms. The first term is the, familiar enough, terms of trade effect: An increase in the international price of the tradeable good increases utility in state i , as long as state i is an exporter of the good in the sense that $X^i > D^i$. The second term reflects the production efficiency of the firm in state i . As long as the marginal cost of producing the tradeable good is below its international price an increase in production increases firm's profits and, therefore, utility. The third term, given by $(\Gamma_g^i - 1) D^i$ reflects the deviation of public good provision in state i from its cost: For given demand D^i , an increase in the tax of state i increases revenues and so public good provision in that state, evaluated by Γ_g^i . The cost of this is the increase, for given international price P , in the domestic price Q^i and so a reduction in consumer surplus by $-D^i$. The fourth term relates, too, to state i public good provision. A change in demand at initial prices changes revenues, valued again at the margin by Γ_g^i . The fifth term, given by $((1/2)\Gamma_G^i - 1) D^i + (1/2)\Gamma_G^i D^j$, reflects the benefit and cost for state i from a change in the federal tax, for given demands in both states (and so for given federal tax base). This term is decomposed as follows: $((1/2)\Gamma_G^i - 1) D^i$ reflects the deviation of federal public good provision from its cost. For given state demand D^i , an increase in the federal tax increases federal revenues by $1/2$ valued, at the margin, by Γ_G^i . As with the state public good, the cost of this is the increase, for given international price P , in the domestic price Q^i and so a reduction in consumer surplus by $-D^i$. The term $(1/2)\Gamma_G^i D^j$ reflects the benefit only to state i (since the consumer of state i consumes the good in that state only) when the federal public good provision increases by $(1/2)D^j$ valued at the margin by the consumer of state i by Γ_G^i . Finally, the last two terms, $(1/2)\Gamma_G^i T_d dD^i + (1/2)\Gamma_G^i T_d dD^j$, reflect the change in the provision of the federal public good when both the demands in state i and state j changes, for a given federal tax T_d , valued again by the consumer in state i by Γ_G^i .

Substituting now (14) and (15) into (17) one expresses the change in utility of state i in terms of the changes in the state and federal taxes given by

$$\begin{aligned}
dv^i &= \frac{\beta^i b}{3} \left[(D^i - 2X^i) + \frac{3}{\beta^i b} (\Gamma_g^i - 1) D^i - \Gamma_g^i t_d^i (2\beta^i + 3\beta^j) - \frac{\Gamma_G T_d}{b} \right] dt_d^i \\
&+ \frac{\beta^j b}{3} \left[(D^i - 2X^i) + \Gamma_g^i t_d^i \beta^i - \frac{\Gamma_G T_d}{b} \right] dt_d^j \\
&- \frac{2}{3} \left[(X^i + D^i) + \Gamma_g^i t_d^i \beta^i + \frac{\Gamma_G T}{2b} - \frac{3\Gamma_G^i (D^i + D^j)}{4} \right] dT_d. \tag{18}
\end{aligned}$$

Notice, for later use, use that the condition for state j is given by

$$\begin{aligned}
dv^j &= \frac{\beta^j b}{3} \left[(D^j - 2X^j) + \frac{3}{\beta^j b} (\Gamma_g^j - 1) D^j - \Gamma_g^j t_d^j (2\beta^j + 3\beta^i) - \frac{\Gamma_G T_d}{b} \right] dt_d^j \\
&+ \frac{\beta^i b}{3} \left[(D^j - 2X^j) + \Gamma_g^j t_d^j \beta^j - \frac{\Gamma_G^j T_d}{b} \right] dt_d^i \\
&- \frac{2}{3} \left[(X^j + D^j) + \Gamma_g^j t_d^j \beta^j + \frac{\Gamma_G^j T}{2b} - \frac{3\Gamma_G^j (D^i + D^j)}{4} \right] dT_d. \tag{19}
\end{aligned}$$

Conditions (18) and (19) are central to the analysis that follows.

We turn now, briefly, to the maximization problem of the state governments and that of the federal. This maximization problem is common throughout the paper—reflecting, of course, the different tax regimes and types of taxes—and so it is not repeated again.

3.1 The maximization problem of the state governments

Each state government cares only about the utility of the consumer that resides within its own jurisdiction. It is assumed that each holds Nash conjectures both against the other but also against the federal government. For given federal tax, the Nash destination based-tax for state i is given by setting the coefficient of dt_d^i in equation (18) equal to zero and solving for t_d^i . In doing so one obtains

$$t_d^i = \Theta^i \left[(D^i - 2X^i) + \frac{3}{\beta^i b} (\Gamma_g^i - 1) D^i - \frac{1}{b} \Gamma_G^i T_d \right], \quad \text{where} \quad \Theta^i \equiv 1/\Gamma_g^i (2\beta^i + 3\beta^j) > 0. \tag{20}$$

Following (19), state j 's tax rate is given by

$$t_d^j = \Theta^j \left[(D^j - 2X^j) + \frac{3}{\beta^j b} (\Gamma_g^j - 1) D^j - \frac{1}{b} \Gamma_G^j T_d \right], \quad \text{where} \quad \Theta^j \equiv 1/\Gamma_g^j (2\beta^j + 3\beta^i) > 0. \tag{21}$$

Notice that, as one would expect, state destination-based taxes depend on whether the state is an importer of the nonnumeraire good, the marginal utility of the state public good (relative to its cost) as well as the level of the federal tax T_d (it does of course depend on the level of state's j commodity tax, but it does so implicitly).

3.2 The maximization problem of the federal government

The objective of the federal government is to maximise, choosing the level of commodity tax T_d , the welfare of the federation taken to be the sum of individual utilities that is, $w = v^i + v^j$.⁹ Following (18) and (19), the federal government commodity tax T_d

⁹One, of course, could introduce weights into the welfare function of the federal government. This, however, will introduce added complications without providing additional insights into issues addressed.

(determined by setting the sum of the coefficients of dT_d equal to zero) is given by

$$T_d = \frac{2b}{\Gamma_G^i + \Gamma_G^j} \left[(D^i + D^j) \left(\left(\frac{3}{4} \Gamma_G^i - 1 \right) + \left(\frac{3}{4} \Gamma_G^j - 1 \right) \right) - (\Gamma_g^i t_d^i \beta^i + \Gamma_g^j t_d^j \beta^j) \right]. \quad (22)$$

3.3 Are state destination-based specific commodity taxes too high or too low?

It is intuitive that the noncooperative behavior on the part of the state governments creates both horizontal and vertical externalities: Horizontal because, as already noted, each government neglects the impact its tax decision has on the other state governments and vertical because each state government unduly discounts the cost of raising its taxes on the federal budget constraint. These externalities, for state i , can be easily seen from equation (18). Suppose for the moment that $dT_d = 0$, and so the federal tax rate is fixed (at its non-cooperative equilibrium value). Following an envelope property (18), then, reduces to

$$dv^i = \frac{\beta^j b}{3} \left[(D^i - 2X^i) + \Gamma_g^i t_d^i \beta^i - \frac{\Gamma_G^i T_d}{b} \right] dt_d^j. \quad (23)$$

Close inspection of (23) shows the two externalities caused by the fiscal behavior of the state j government.

The magnitude $(D^i - 2X^i) + \Gamma_g^i t_d^i \beta^i$ in the square brackets in (23) consists of three horizontal externalities, caused by the fact that state j government's tax behavior ignores its impact on the welfare of state i .

- The first horizontal externality, given by $D^i - X^i$, is through the terms of trade. In raising taxes, state j affects the international price of the tradeable good (to the magnitude of $\beta^j b/3$) and, to the extent that state i is an importer of the good, and so $D^i - X^i > 0$, state j 's tax is too low in equilibrium, from the point of view of state i . An increase in the tax of state j increases welfare of state i . This is not, however, true for state j . For state j an exporter of the tradeable good and, hence, state i 's tax is too high in equilibrium.¹⁰ There is, thus, a conflict of inefficiencies arising from the strategic incentive each state has to affect the terms of trade.
- The second horizontal externality concerns the change in profit income of the consumer in state i , a change that state j ignores when setting taxes. For given deviation of the international price from the cost of the tradeable good, which is equal, in equilibrium to—following (9)— bX^i , a change in the state j 's tax rate affects production in state i via the international price—to the magnitude of $-\beta^j/3$ —of the tradeable good. This, of course, is applicable to state j , too. This externality points towards state taxes that are too high in equilibrium.

¹⁰In this case an increase in state i 's tax will have a negative effect, since $D^j - X^j < 0$, to the magnitude of $\beta^j b/3$.

- The third horizontal externality impacts through tax revenues. In setting the commodity tax, state j ignores the loss of revenues incurred to state i (loss that is valued by Γ_g^i), since an increase in the tax rate in state j affects the demand of the tradeable good in state i through the change in the international price of that good. The magnitude of this externality is $t_d^i \beta^j b \beta^i / 3$ and, following from $t_d^i > 0$, is strictly positive. This is also true for the tax setting behavior of state i . It is, then, clear that this externality points towards state taxes that are too low in equilibrium.

The vertical externality reflects the change in the federal public good provided to state i government caused by the tax setting behavior of the state j government. The magnitude of this externality is $-T_d(\beta^j/3)$,¹¹ and is valued by the consumer of state i by Γ_G^i . This externality points towards state taxes that are too high in equilibrium.

3.3.1 Are state destination-based commodity specific taxes too high or too low?

It is so clear that whether state destination-based commodity taxes are too low or too high, conditional upon the federal tax rate, depends on the balance of the externalities identified in the preceding paragraph. At this level of generality (inherent in an asymmetric equilibrium) one can construct examples, based on reasonable parameter values, such there is under/over-taxation at the state level, but it is difficult to identify conditions under which one externality dominates the other. This difficulty arises by the asymmetry in preferences and marginal valuation of public goods. To make some progress we remove this dependency by paying attention to an equilibrium in which the responsiveness of demand in both state is the same,¹² and so $\beta^i = \beta^j = \beta$, and both states have the same marginal valuation for both the state public good—in the sense that $\Gamma_g^i = \Gamma_g^j \equiv \Gamma_g$ —and for the federal public good and so $\Gamma_G^i = \Gamma_G^j \equiv \Gamma_G$.

To evaluate the extend of tax inefficiencies at state level first denote the aggregate level of welfare by

$$w(t_d^i + T_d, t_d^j + T_d, T_d) = v^i(t_d^i + T_d, t_d^j + T_d, T_d) + v^j(t_d^i + T_d, t_d^j + T_d, T_d) . \quad (24)$$

Perturbation now of (24) gives the sum of the terms that appear in (18) and (19) which—upon evaluation at the non-cooperative equilibrium, using (20)-(22), and for $dt^i = dt^j = dt$ —reduce to

$$dw = (\Gamma_g - \Gamma_G) (D^i + D^j) dt_d . \quad (25)$$

¹¹This is derived as follows. Perturb (6) to obtain $dG = (1/2)T_d(dD^i/dt^j + dD^j/dt^j)$ which upon making use of (1) becomes $dG = (1/2)T_d(-dP/dt^j(\beta^i + \beta^j) - \beta^j)$. Upon simplifying, the latter expression, using (14), one arrives at $dG = T_d(-\beta^j/3)dt_d^j$.

¹²This, of course, does not imply that demands for the tradeable good are the same in both states. This is because we maintain that $\alpha^i \neq \alpha^j$ and $c^i \neq c^j$.

Condition (25) simply says that the direction of inefficiencies depend upon the discrepancy of the marginal valuation of the federal to the state public good. This is intuitive. For in this case the federal tax, as an envelope property, has no effect on the aggregate level of welfare and so the federal tax does not cause any externalities to state welfare. What is left, then, are the externalities arising from the tax setting behavior of the state governments, externalities that are—to a large extent—corrected by the federal tax. To see this notice that in (25) the externalities through the terms of trade and profits (identified after (23)) disappear leaving only those arising through public good provision. Summarizing :

Proposition 1 *In the presence of destination-based commodity taxes, if the federal public is more (less) valued than the state public good then state government taxes are too high (low) in equilibrium from a welfare point of view.*

3.3.2 A closer look at the horizontal externalities

One might be tempted to think that if there is no federal government, so $\Gamma_G = 0$ in (25), state taxes will be too low in equilibrium. Interestingly, however, this is not the case. For, if there is no federal government, there is no implicit internalization of horizontal externalities come about the implicit consideration of the federal tax by the state governments, even though the federal government has Nash conjectures *vis à vis* state governments. To see this suppose that there is no federal government and so $T_d = 0$ and $\Gamma_G = 0$. In this case the Nash equilibrium level of taxes for state i and state j are given,, following (20) and (21), respectively by

$$t_d^{jN} = \Theta^i \left[(D^i - 2X^i) + \frac{3}{\beta^i b} (\Gamma_g^i - 1) D^i \right], \quad \text{where} \quad \Theta^i \equiv 1/\Gamma_g^i (2\beta^i + 3\beta^j), \quad (26)$$

$$t_d^{jN} = \Theta^j \left[(D^j - 2X^j) + \frac{3}{\beta^j b} (\Gamma_g^j - 1) D^j \right], \quad \text{where} \quad \Theta^j \equiv 1/\Gamma_g^j (2\beta^j + 3\beta^i). \quad (27)$$

It is clear from the analysis in the preceding subsection that in the absence of the federal government and asymmetries in the responsiveness in the demand for the tradeable good the direction of inefficiency state taxes will depend upon the externalities through profits and tax revenues. To see this notice that perturbation of (24) gives, following (23) and its counterpart for state j ,

$$dw = \frac{\beta^j b}{3} [(D^i - 2X^i) + \Gamma_g^i t_d^i \beta^i] dt_d^j + \frac{\beta^i b}{3} [(D^j - 2X^j) + \Gamma_g^j t_d^j \beta^j] dt_d^i. \quad (28)$$

To see the reason that the externality through the terms of trade disappears notice that with $\beta^i = \beta^j \equiv \beta$ the loss of welfare for an importing state, due an increase in the international price of the tradeable good, is equal to the welfare gain for the exporting state. This follows from the fact that in (28), with $dt^i = dt^j$, the terms $(\beta^j b/3) (D^i - X^i) + (\beta^i b/3) (D^j - X^j)$, become, $(1/6) [(D^i - X^i) + (D^j - X^j)] = 0$, with

the equality following from (3). The only externalities thus appear through profits and public good provision. To see this notice that the impact of state j 's taxation on the profits of state i is given by $-\beta^j b X^i / 3$ which, following $\beta^i = \beta^j \equiv \beta$, is equal to $-(1/6)X^i$. Adding to this the impact of state i 's taxation on the welfare of state j one obtains the total welfare loss, through profits, for both states which is equal to $-(1/6)(X^i + X^j)$. Turning now to the externalities through tax revenues one will recall that the impact of state j 's taxation on the tax revenues of state i is given by $\Gamma_g^i t_d^i \beta^j b \beta^i / 3$ which, following $\beta^i = \beta^j \equiv \beta$ and $\Gamma_g^i \equiv \Gamma_g$, is equal to $\Gamma_g t_d^i \beta / 6$. Adding the corresponding welfare loss of state j , caused by the tax setting behavior of state i , after using (26) and (27), one obtains $(1/30)(X^i + X^j)[-1 + 6(\Gamma_g - 1)]$.

Adding now the losses from the two externalities together, after using (3), one obtains $dw = (1/5)[(\Gamma_g - 2)(D^i + D^j)] dt$. It is so clear that in this case state taxes can be either too low or too high depending on the value of the marginal utility of the public good.¹³ Summarizing:

Proposition 2 *Under destination-based commodity taxes, and in the absence of vertical tax externalities, state taxes are too high (low) if and only if $\Gamma_g > 2$ ($\Gamma_g < 2$).*

We turn to the origin principle of taxation.

4 Origin principle and specific taxation

The background of the analysis is similar so we keep the description brief. It will be recalled that under the origin principle of taxation taxes are levied and revenues collected by the state state government in which the good is produced. International arbitrage, in this case, equates consumer prices across the two countries, and so $Q^i = Q^j = Q$. Denoting t_o^i (t_o^j) the specific origin tax of state i (j) the firm i 's profit function is

$$\Pi^i = (Q - t_o^i - T_o - c^i)X^i - F^i \quad ; \quad \Pi^j = (Q - t_o^j - T_o - c^j)X^j - F^j \quad , \quad (29)$$

with necessary condition

$$Q - t_o^i - T_o - c^i = bX^i \quad ; \quad Q - t_o^j - T_o - c^j = bX^j \quad . \quad (30)$$

Making use of (1) into (3) we find the aggregate inverse demand function given by

$$Q = b[a - (X^i + X^j)] \quad , \quad \text{where} \quad a \equiv \alpha^i + \alpha^j \quad , \quad b \equiv 1/(\beta^i + \beta^j) \quad . \quad (31)$$

The revenues from taxes are used to provide public goods

$$g^i = t_o^i X^i \quad ; \quad g^j = t_o^j X^j \quad ; \quad G = (1/2)T(X^i + X^j) \quad . \quad (32)$$

¹³This, of course, is still valid in a symmetric equilibrium. In this case with $D^i = D^j$, $X^i = X^j$, the change in welfare is given by $dw = (2/5)(\Gamma_g - 2)Ddt$.

Solving simultaneously (30), (31) one obtains

$$Q = (1/3) [b a + (t_o^i + T_o) + (t_o^j + T_o) + c^i + c^j] , \quad (33)$$

$$X^i = (1/3b) [b a - 2t_o^i + t_o^j - T_o - 2c^i + c^j] , \quad (34)$$

$$X^j = (1/3b) [b a + t_o^i - 2t_o^j - T_o - 2c^j + c^i] . \quad (35)$$

Notice, for later use, that

$$dQ = (1/3) [dt_o^i + dt_o^j + 2dT_o] , \quad (36)$$

$$dX^i = -(1/3b) [2dt_o^i - dt_o^j + dT_o] , \quad (37)$$

$$dX^j = -(1/3b) [-dt_o^i + 2dt_o^j + dT_o] . \quad (38)$$

The change in utility in state i is then given by

$$\begin{aligned} dv^i &= (X^i - D^i) dQ + (Q - t_o^i - T - c^i) dX^i + (\Gamma_g^i - 1) X^i dt_o^i + \Gamma_g^i t_o^i dX^i \\ &+ [((1/2)\Gamma_G^i - 1) X^i + (1/2)\Gamma_G^i X^j] dT_o + \Gamma_G^i T_o dX^i + \Gamma_G^i T_o dX^j . \end{aligned} \quad (39)$$

Here too, there are seven effects that impact upon the utility of the consumer in state i . Since we explore these after equation (17) we do not repeat the discussion here. The interpretation of these effects under the destination principle carries over to the origin principle too, with, of course, the appropriate interpretation of the variables.

In this case, state i government origin-based tax is given by

$$t_o^i = \Theta^i \left(-4X^i - D^i + 3\Gamma_g^i X^i - \frac{\Gamma_G^i T_o}{2b} \right), \quad \text{where } \Theta^i \equiv b/2\Gamma_g^i > 0 . \quad (40)$$

An analogous expression holds for state j government. For later use, the j 's government tax is given by

$$t_o^j = \Theta^j \left(-4X^j - D^j + 3\Gamma_g^j X^j - \frac{\Gamma_G^j T_o}{2b} \right), \quad \text{where } \Theta^j \equiv b/2\Gamma_g^j > 0 . \quad (41)$$

As with the destination-based taxes, the federal government provides a public good in both states, financed by a specific tax levied on world production. It therefore choose T_o to maximize the federation welfare given by $w = v^i + v^j$. In doing so it sets¹⁴

$$T_o = \frac{b}{2\Gamma_G} \left[(X^i + X^j) (3\Gamma_G - 4) - \frac{\Gamma_g^i t_o^i}{b} - \frac{\Gamma_g^j t_o^j}{b} \right] . \quad (42)$$

¹⁴Notice that (42) assumes that $\Gamma_G^i = \Gamma_G^j \equiv \Gamma_G$.

4.1 Horizontal and vertical externalities under origin-based commodity specific taxes

Perturbing welfare for state i , given by $v^i(t_o^i + T_o, t_o^j + T_o, T_o)$, and using (40), one obtains

$$dv^i = \frac{1}{3} \left[(2X^i - D^i) + \frac{\Gamma_g^i t_o^j}{b} - \frac{\Gamma_G T_o}{2b} \right] dt_o^j. \quad (43)$$

An analogue expression holds for the change in the welfare of state j .

As with the destination principle, (43) is central to the analysis. Close inspection of (43) reveals that, here too, there are two externalities caused by the tax behavior of the state j government: The horizontal and the vertical. But the mechanism here is different to the one under the destination principle.

The magnitude $2X^i - D^i + \Gamma_g^i t_o^j / b$ in the square brackets in (43) consists of three horizontal externalities too, caused by the fact that state j government's tax behavior ignores its impact on the welfare of state i .

- The first horizontal externality, given by $-(D^i - X^i)$, is through the terms of trade. In raising taxes, state j affects the international price of the tradeable good (to the magnitude of $1/3$) and, to the extent that state i is an importer of the good, and so $D^i - X^i > 0$, state j 's tax is too high in equilibrium, from the point of view of state i . This is not, however, true for state j . For this state is an exporter of the tradeable good and, hence, state i 's tax is too low in equilibrium.¹⁵ There is, thus, here too, a conflict of inefficiencies arising from the strategic incentive each state has to affect the terms of trade.
- The second horizontal externality is the change in profit income of the consumer in state i , a change that state j ignores when setting its tax. For given deviation of the international price from the cost of the tradeable good, which is equal, in equilibrium to—following (30)— bX^i , a change in the state j 's tax rate affects production in state i via the international price—to the magnitude of $1/3$ —of the tradeable good. This, of course, is applicable to state j too. This points towards taxes that are too low in equilibrium.
- The third horizontal externality is through public good provision. In setting its tax state j ignores the the loss of revenues, valued by Γ_g^i , incurred to state i , since an increase in the tax rate in state j affects the supply of the tradeable good in i via the change in its international price, the magnitude of which is $1/3b$. This points towards state taxes that are too low in equilibrium.

¹⁵In this case an increase in state i 's tax will have a negative effect, since $X^j - D^j < 0$, to the magnitude of $1/3$.

The vertical externality reflects the change in the federal public good provided to state i government due to the tax setting behavior of the state j government and its magnitude is given by $T_d(-1/6b)$.¹⁶ This externality points towards state taxes that are too high in equilibrium.

4.1.1 Are origin-based commodity specific state taxes too high or too low?

It is so clear that whether state origin-based commodity taxes are too low or too high, conditional upon the federal tax rate, depends on the balance of the externalities identified in the preceding paragraph. As with the destination-based taxes, one can construct examples, based on reasonable parameter values, such there is under/over-taxation at the state level, but it is difficult to identify conditions under which one externality dominates the other. To make some progress we remove this dependency by paying attention to an equilibrium in which the responsiveness of demand in both state is the same, and so $\beta^i = \beta^j = \beta$ and also both states have the same marginal valuation for both the state and federal public goods in the sense that $\Gamma_g^i = \Gamma_g^j \equiv \Gamma_g$ and $\Gamma_G^i = \Gamma_G^j \equiv \Gamma_G$.

Perturbing welfare, denoted by $w(t_o^i + T_o, t_o^j + T_o, T_o)$, for $dt_o^i = dt_o^j \equiv dt_o$, and evaluating at the non-cooperative equilibrium, using (40)-(42) one arrives at

$$dw = (\Gamma_g - \Gamma_G) (X^i + X^j) dt_o . \quad (44)$$

We thus have:

Proposition 3 *In the presence of origin-based commodity taxes, if the federal public is more (less) valued than the state public good then state government taxes are too high (low) in equilibrium.*

A simple comparison of Propositions 1 and 3 reveals that under both principles the direction of inefficiencies critically depends on the the marginal valuation of the state public good *vis à vis* the federal public good. To emphasize this:

Corollary 1 *The conditions under which taxes are too high or too low in equilibrium, are the same under both destination and origin principle of taxation.*

4.1.2 A closer look at the horizontal externalities

We now take a closer look at horizontal externalities. To do this we set $T = 0$ and $\Gamma_G = 0$ in (40) and (41) to obtain

$$t_o^i = (b/2\Gamma_g^i) (-4X^i - D^i + 3\Gamma_g^i X^i) \quad ; \quad t_o^j = (b/2\Gamma_g^j) (-4X^j - D^j + 3\Gamma_g^j X^j) . \quad (45)$$

¹⁶This is derived as follows. Perturb (32) to obtain $dG = (1/2)T_o(dX^i/dt_o^j + dX^j/dt_o^i)$ which upon making use of (37) and (38) becomes $dG = -(1/3)T_o(1/2b)dt_o^j$.

Substituting (45) into (43), and into the counterpart for state j , one obtains

$$dv^i = (1/2) (-D^i + \Gamma_g^i X^i) dt_o^j \quad ; \quad dv^j = (1/2) (-D^j + \Gamma_g^j X^j) dt_o^i, \quad (46)$$

which implies that

$$dw = (1/2) [(\Gamma_g - 1) (X^i + X^j)] dt_o. \quad (47)$$

(47) simply says that a coordinated increase in state taxes unambiguously increases total welfare if and only if $\Gamma_g > 1$. It could reduce welfare in the arguably unlikely case that $\Gamma_g < 1$. So in equilibrium consumers are under-taxed, that is, starting from the Nash equilibrium level of taxes, welfare would be increased by a rise on the states' taxes. To emphasize:

Proposition 4 *Under origin-based commodity taxes, and in the absence of vertical tax externalities, state taxes are too high (low) if and only if $\Gamma_g < 1$ ($\Gamma_g > 1$).*

Close inspection of Propositions 2 and 4 reveals an interesting contrast between destination-based and origin-based commodity taxes.

We turn now to a discussion of ad valorem taxation. We keep the discussion brief.

5 Destination principle and ad valorem taxation

Under the destination principle the price of the tradeable good, denoting the ad valorem tax by the subscript a , is given by

$$Q^i = P(1 + t_{da}^i + T_{da}) \quad ; \quad Q^j = P(1 + t_{da}^j + T_{da}), \quad (48)$$

where P is, as before, the international price of the tradeable good.

Public good provision by the state i (j) government is given by

$$g^i = t_{da}^i P D^i \quad ; \quad g^j = t_{da}^j P D^j, \quad (49)$$

whereas the federal government provides each state with tax receipts, given by

$$G = (1/2) T_{da} P (D^i + D^j). \quad (50)$$

Making use of (48) in (3) one obtains the inverse aggregate demand given by

$$P = (a - X^i - X^j) / \delta, \quad (51)$$

where, it will be recalled from (31), $a \equiv \alpha^i + \alpha^j$, and

$$\delta \equiv \beta^i (1 + t_{da}^i + T_{da}) + \beta^j (1 + t_{da}^j + T_{da}). \quad (52)$$

Profit maximization in states i and j , respectively, gives

$$X^i = \delta(P - c^i) \quad ; \quad X^j = \delta(P - c^j) . \quad (53)$$

Making use of (51) and (53) one obtains

$$P = (1/3)[a/\delta + c^i + c^j] , \quad (54)$$

$$X^i = (1/3)[a + \delta(c^j - 2c^i)] , \quad (55)$$

$$X^j = (1/3)[a + \delta(c^i - 2c^j)] . \quad (56)$$

The tax setting behavior of the state governments cause fiscal externalities. Relegating the details to Appendix A, the externalities can be shown to be the following.

- The first horizontal externality, given by $(1/P)(D^i Q^i - X^i P)$, is, loosely speaking, through the terms of trade. In raising taxes state j affects negatively the equilibrium price of the tradeable good (to the magnitude of $\beta^j a / (3\delta^2)$), and so to the extent that the value of demand, evaluated at price Q^i , is higher than the value of production X^i , evaluated at the international price P , then state j 's destination-based ad valorem tax is too low in equilibrium. An increase in the tax of state j will increase welfare in state i .
- The second horizontal externality is through profits. For given deviation of the international price from the cost of producing the tradeable good $P - c^i$, equal in equilibrium to X^i / δ , a change in the state j 's tax affects production in state i via the international price—to the magnitude of $\beta^j (c^j - 2c^i) / 3$. This implies that the direction of inefficiency depends on the relative efficiency of the firm in state i relative to the firm in state j . In particular, if $c^j > 2c^i$ ($c^j < 2c^i$) then state j tax is too low (high) from the point of view of state i . But if $c^j = 2c^i$ then this externality disappears for state i .¹⁷
- The third horizontal externality is through tax revenues. As noted earlier, taxes affect revenues through two channels. The first one is, for given demand D^i , through the international price: An increase in the tax rate in state j reduces the international price of the tradeable good thereby reducing tax revenues by $-t_{da}^i a \beta^j D^i / (3\delta^2)$. The second is through demand D^i . An increase in the tax rate of the tradeable good in state j , reduces the international producer price and therefore increases demand in state i thereby increasing tax revenues there by $t_{da}^i \beta^i a \beta^j Q^i / (3\delta^2)$. Consequently, whether this externality points to too high taxes, or too low, depends on the balance of this two effects.

¹⁷It is easy to verify that with $c^j = 2c^i$, state i tax is too high for state j .

The vertical externality caused, too, by the behavior of state j government, is composed of the following terms.

- An increase in the tax rate in state j , changes federal expenditure allocated to state i through, as before, a change in demand in state i —the magnitude of which is $\beta^i a \beta^j Q^i / (3\delta^2 P)$, and for given demand in state i , through the change in the international price and through the change in demand in state j —the magnitude of which is $(\beta^j)^2 a Q^i / (3\delta^2 - \beta P)$. For given initial international price, P , total federal public good provision changes to the magnitude of $-T_{da}(D^i + D^j)\beta^j a / (6\delta^2)$.

It is thus clear that with ad valorem taxation the sign and direction of externalities differ to the ones under specific taxation. Two observations are in order here. Firstly, the sign of the horizontal externality through profits, under ad valorem taxes, depends on the relative efficiency of the imperfectly competitive firms, a dependence that does not appear under specific taxation. It is, thus, the case that under specific taxation this externality points to state taxes that are too low, from a welfare point of view, whereas under ad valorem taxation it may be too high or too low. Secondly, the sign of the horizontal externality through tax revenues under ad valorem taxes is ambiguous in sign and therefore this externality may point to states taxes that are too high or too low from a welfare point of view. This is not, however, the case under specific taxation. In this case this externality is always positive pointing towards state taxes that are too low. This latter difference arise, of course, from the fact that the dependence of tax revenues on the international price of the tradeable good, a dependence that appears here both indirectly, through the demand for the tradeable good, $Q^i = P(1 + t_{da}^i + T_{da})$, but also directly since the tax is directly applicable to the value of consumption PD^i .

We have just seen that whether the state i 's tax is too low or too high depends on the balance of externalities. It is possible to construct simple examples that show that this balance can go either way. Consider, for instance, the following.¹⁸

Example 1. Ad valorem destination-based taxes and over-taxation at the state level: Suppose that $\alpha^i = 10$, $\alpha^j = 12$, $\beta^i = \beta^j = 0.6$, $c^i = 3$, $c^j = 4$, $\Gamma_g^i = \Gamma_g^j = 1.5$ and $\Gamma_G^i = \Gamma_G^j = 1.8$. Based on these parameter values it is the case that $dw = dv^i/dt_{da}^j + dv^j/dt_{da}^i = -1.112 < 0$, and so taxation at the state level is too high: Both states would gain in welfare by strictly reducing taxes.¹⁹

Example 2. Ad valorem destination-based taxes and under-taxation at the state level: Suppose that $\alpha^i = 13$, $\alpha^j = 12$, $\beta^i = \beta^j = 1.8$, $c^i = 3$, $c^j = 4$, $\Gamma_g^i =$

¹⁸The details behind the examples are available upon request. All computation have been performed using MAPLE v.9.

¹⁹These parameter values give $t_{da}^{iN} = 0.045$, $t_{da}^{jN} = 0.329$, $T^N = 0.758$, $X^i = 5.777$, $X^j = 3.442$, $D^i = 4.075$, $D^j = 5.144$, and $P = 5.475$.

$\Gamma_g^j = 2.5$ and $\Gamma_G^i = \Gamma_G^j = 2.4$. Based on these parameter values it is the case that $dw = dv^i/dt_{da}^j + dv^j/dt_{da}^i = 5.607 > 0$, and so taxation at the state level is too low: Both states would gain in welfare by strictly increasing taxes.²⁰

We turn now to the origin principle and ad valorem taxation.

6 Origin principle and ad valorem taxation

International arbitrage equates consumer prices across the jurisdictions, and so $Q^i = Q^j = Q$. Denoting t_{oa}^i (t_{oa}^j) the ad valorem origin tax of state i (j) then

$$P^i(1 + t^i + T) = P^j(1 + t^j + T) . \quad (57)$$

The aggregate inverse demand function is still given by (31) whereas the firm i 's profit function is

$$\Pi^i = (Q/(1 + t_{oa}^i + T_{oa}) - c^i)X^i - F^i \quad ; \quad \Pi^j = (Q/(1 + t_{oa}^j + T_{oa}) - c^j)X^j - F^j , \quad (58)$$

with necessary condition, following from (31),

$$Q/(1 + t_{oa}^i + T_{oa}) - c^i = bX^i/(1 + t_{oa}^i + T_{oa}) \quad ; \quad Q/(1 + t_{oa}^j + T_{oa}) - c^j = bX^j/(1 + t_{oa}^j + T_{oa}) . \quad (59)$$

The revenues from taxes are used to provide public goods

$$g^i = t_{oa}^i P^i X^i \quad ; \quad g^j = t_{oa}^j P^j X^j \quad ; \quad G = (1/2)T(P^i X^i + P^j X^j) . \quad (60)$$

Solving simultaneously (31) and (59), one obtains

$$Q = (1/3) [b a + c^i(1 + t_{oa}^i + T_{oa}) + c^j(1 + t_{oa}^j + T_{oa})] , \quad (61)$$

$$X^i = (1/3b) [b a + c^j(1 + t_{oa}^j + T_{oa}) - 2c^i(1 + t_{oa}^i + T_{oa})] , \quad (62)$$

$$X^j = (1/3b) [b a + c^i(1 + t_{oa}^i + T_{oa}) - 2c^j(1 + t_{oa}^j + T_{oa})] . \quad (63)$$

As before the tax setting behavior of state, say j , creates externalities. These externalities (for brevity the details are relegated to Appendix B) are the following:

- The first horizontal externality, given by $(1/Q)(X^i P^i - D^i Q)$, is, loosely speaking, through the terms of trade. In raising taxes state j affects the equilibrium price of the tradeable good to the magnitude of $c^j/3$, and so to the extent that the value of production, evaluated at price P^i , is higher than the value of demand D^i , evaluated at the international price Q , then state j 's origin-based ad valorem tax is too low in equilibrium. An increase in the tax of state j will increase welfare in state i .

²⁰These parameter values give $t_{da}^{iN} = 0.258$, $t_{da}^{jN} = 0.236$, $T^N = 0.057$, $X^i = 5.204$, $X^j = 0.509$, $D^i = 3.273$, $D^j = 2.439$, and $P = 4.108$.

- The second horizontal externality is through profits. For given deviation of the international price from the cost of producing the tradeable good $P^i - c^i$, equal in equilibrium to $bX^i/(1 + t_{oa}^i + T_{oa})$, a change in the state j 's tax affects production in state i —to the magnitude of $c^j/(3b)$. This points towards taxes that are too low in equilibrium. It is applicable, of course, to state j too.
- The third horizontal externality is through tax revenues. As noted earlier, taxes affect revenues through two channels. The first one is, for given supply X^i , through the international price: An increase in the tax rate in state j increases the international price of the tradeable good thereby increasing tax revenues by $t_{oa}^i X^i c^j / (3(1 + t_{oa}^i + T_{oa}))$. The second is through supply X^i . An increase in the tax rate of the tradeable good in state j , increases the international price and therefore increases supply in state i thereby increasing tax revenues there by $t_{oa}^i Q c^j / (3b)$. Consequently, this externality points to a tax that is too low.

The vertical externality caused, too, by the behavior of state j government, has the following components.

- An increase in the tax rate in state j , changes federal expenditure allocated to state i through, as before, a change in supply of the tradeable good in state i —the magnitude of which is $c^j/(3b)$, and for given demand in state i , through the change in the international price and through the change in supply in state j —the magnitude of which is $-2c^j/(3b)$. All in all, in this case, federal tax revenues allocated to state i change by

$$-\frac{T_{oa} Q c^j}{6b} \left[\frac{(1 + T_{oa}) + (2t_{oa}^i - t_{oa}^j)}{(1 + t_{oa}^i + T_{oa})(1 + t_{oa}^j + T_{oa})} \right], \quad (64)$$

and so is ambiguous.

- For given supply in both states X^i and X^j , the change in the state j 's tax also affects total federal public good provision via the change in the international price Q . This change is in the magnitude of

$$\frac{T_{oa} c^j}{6} \left(\frac{X^i}{1 + t_{oa}^i + T_{oa}} + \frac{X^j}{1 + t_{oa}^j + T_{oa}} \right) > 0, \quad (65)$$

and so this externality points towards taxes that are too low (the inequality follows from $t_{oa}^i, t_{oa}^j, T_{oa} > 0$).

- Finally, for given international price of the tradeable good Q , the change in the state j 's tax also affects the producer prices P^i thereby changing federal revenues by

$$\frac{T_{oa} X^j Q}{2(1 + t_{oa}^j + T_{oa})^2} > 0. \quad (66)$$

This externality, too, is positive pointing to state j tax that is too low from the point of view of state i 's welfare.

Consequently, it follows that whether state j 's tax on the tradeable commodity is too low or too high depends on the balance of these externalities.

It is possible to construct simple examples that show that the balance can go either way. Consider, for instance, the following.

Example 1. Ad valorem origin-based taxes and over-taxation at the state level: Suppose that $\alpha^i = 10$, $\alpha^j = 12$, $\beta^i = \beta^j = 1$, $c^i = 3$, $c^j = 4$, $\Gamma_g^i = \Gamma_g^j = 1.4$ and $\Gamma_G^i = \Gamma_G^j = 1.5$. Based on these parameter values it is the case that $dw = dv^i/dt_{do}^j + dv^j/dt_{do}^i = -2.745 < 0$, and so taxation at the state level is too high: Both states would gain in welfare by strictly reducing taxes.²¹

Example 2. Ad valorem origin-based taxes and under taxation at the state level: Suppose that $\alpha^i = 13$, $\alpha^j = 12$, $\beta^i = \beta^j = 1.8$, $c^i = 3$, $c^j = 4$, $\Gamma_g^i = \Gamma_g^j = 1.8$ and $\Gamma_G^i = \Gamma_G^j = 2.4$. Based on these parameter values it is the case that $dw = dv^i/dt_{do}^j + dv^j/dt_{do}^i = 1.804 > 0$, and so taxation at the state level is too high: Both states would gain in welfare by strictly increasing taxes.²²

Close inspection of the externalities under origin-based specific and ad valorem taxation shows that the horizontal ones (terms of trade, profits and tax revenues) have the same direction. The significant difference exists in the consideration of the vertical externality. While under specific taxation it is always negative, pointing to state taxes that are too high from a welfare point of view, it is ambiguous when taxes are ad valorem.

7 Summary and concluding remarks

The main objective of this paper has been to identify the fiscal externalities arising from two-types-two-regimes commodity tax competition between the two levels of government in a federal system when firms have market power. In particular, the framework has considered two types of taxes, specific and ad valorem, that can be levied on a destination or on an origin basis. It has been shown that the direction of inefficiencies when both levels of government are present, under specific taxation, is not affected by the tax principle in place but it does *critically* depend on the deviation of the marginal valuation for the federal public good from the marginal valuation for the state public good. In the equilibrium considered, state taxes have been shown to be too low (high) from an

²¹These parameter values give $t_{oa}^{iN} = 0.409$, $t_{oa}^{jN} = 0.05$, $T_{oa}^N = 0.572$, $X^i = 3.733$, $X^j = 2.644$, $D^i = 2.188$, $D^j = 4.188$, and $Q = 7.811$.

²²These parameter values give $t_{oa}^{iN} = 0.2579$, $t_{oa}^{jN} = 0.015$, $T_{oa}^N = 0.299$, $X^i = 3.437$, $X^j = 1.314$, $D^i = 2.876$, $D^j = 1.876$, and $Q = 5.625$.

equilibrium point of view, under both destination and origin-based taxes, if consumer value the state more (less) than the federal public good.

Things, however, are more complicated under ad valorem taxation. This is perhaps not surprising given the (non-additive) dependence of the market on prices. Nevertheless the analytics have shown that with ad valorem taxation the externalities in a federal system, though easy to be identified, they are cumbersome to be evaluated. For such an evaluation a precise knowledge of the fundamental parameters of the demand and supply functions are required as well as the preferences of the consumers for the public goods.

It has also been shown that with ad valorem taxation the sign and direction of externalities differ to the ones under specific taxation in two respects. Firstly, the sign of the horizontal externality through profits, under ad valorem taxes, depends on the relative efficiency of the imperfectly competitive firms, a dependence that does not appear under specific taxation. It is, thus, the case that under specific taxation this externality points to state taxes that are too low, from a welfare point of view, whereas under ad valorem taxation it may be too high or too low. Secondly, the sign of the horizontal externality through tax revenues under ad valorem taxes is ambiguous in sign and therefore this externality may point to states taxes that are too high or too low from a welfare point of view. This is not, however, the case with origin-based taxes. In this case specific and ad valorem taxation deliver horizontal externalities of the same direction (terms of trade, profits and tax revenues). The significant difference exists in the consideration of the vertical externality.

APPENDICES

APPENDIX A

Derivation of externalities under the destination principle and ad valorem taxes.

Perturbing (54)-(56) gives

$$dP = [-a/(3\delta^2)] [\beta^i dt_{da}^i + \beta^j dt_{da}^j + (1/b)dT_{da}] , \quad (\text{A.1})$$

$$dX^i = [(c^j - 2c^i)/3] [\beta^i dt_{da}^i + \beta^j dt_{da}^j + (1/b)dT_{da}] , \quad (\text{A.2})$$

$$dX^j = [(c^i - 2c^j)/3] [\beta^i dt_{da}^i + \beta^j dt_{da}^j + (1/b)dT_{da}] . \quad (\text{A.3})$$

Equations (A.2) and (A.3) reveal that, interestingly, that whether production of the tradeable good diminishes or not in state i and j , respectively—as a response to an increase in the destination-based ad valorem taxes—depends on the relative efficiency of production of the tradeable good in states i and j .

Total differentiation of equation (16), making use of the differentials in (49) and (50), yields

$$\begin{aligned} dv^i &= (1/P) (X^i P - D^i Q^i) dP + (P - c^i) dX^i \\ &+ (\Gamma_g^i - 1) PD^i dt_{da}^i + \Gamma_g^i t_{da}^i (PdD^i + D^i dP) \\ &+ [((1/2)\Gamma_G^i - 1) PD^i + (1/2)\Gamma_G^i PD^j] dT_{da} + (\Gamma_G^i/2) T_{da} P dD^i + (\Gamma_G^i/2) T_{da} P dD^j \\ &+ T_{da}(\Gamma_G^i/2)(D^i + D^j)dP . \end{aligned} \quad (\text{A.4})$$

Substituting the differentials in (A.2) and (A.3), and those of (49) and (50), in (A.4) one arrives at the externalities identified in the text. \square

APPENDIX B

Derivation of externalities under the origin principle and ad valorem taxes.

Perturbing (61)-(63) one obtains

$$dQ = (1/3) [c^i dt_{oa}^i + c^j dt_{oa}^j + (c^i + c^j)dT_{oa}] , \quad (\text{B.1})$$

$$dX^i = (1/3b) [c^j dt_{oa}^j - 2c^i dt_{oa}^i + (c^j - 2c^i)dT_{oa}] , \quad (\text{B.2})$$

$$dX^j = (1/3b) [c^i dt_{oa}^i - 2c^j dt_{oa}^j + (c^i - 2c^j)dT_{oa}] . \quad (\text{B.3})$$

Perturbing now (16) gives the change in utility in state i , given by

$$\begin{aligned}
dv^i &= (1/Q) (X^i P^i - D^i Q) dQ + (Q/\tau^i - c^i) dX^i - \left(X^i Q / (\tau^i)^2 \right) (dt_{oa}^i + dT_{oa}) \quad (\text{B.4}) \\
&+ \Gamma_g^i (t_{oa}^i X^i / \tau^i) dQ + \Gamma_g^i \left(Q(1 + T_{oa}) X^i / (\tau^i)^2 \right) dt_{oa}^i \\
&+ \Gamma_g^i (t_{oa}^i Q / \tau^i) dX^i - \Gamma_g^i (t_{oa}^i Q X^i / (\tau^i)^2) dT_{oa} \\
&+ \Gamma_G^i / (2\tau^i) [T_{oa} Q dX^i + (Q X^i (1 + t_{oa}^i) / \tau^i) dT_{oa} + T_{oa} X^i dQ - (T_{oa} X^i Q / \tau^i) dt_{oa}^i] \\
&+ \Gamma_G^i / (2\tau^j) [T_{oa} Q dX^j + (Q X^j (1 + t_{oa}^j) / \tau^j) dT_{oa} + T_{oa} X^j dQ - (T_{oa} X^j Q / \tau^j) dt_{oa}^j],
\end{aligned}$$

where $\tau^k \equiv 1 + t_{oa}^k + T_{oa}$, $k = i, j$. (An analogous condition applies to state j .)

Substituting the differentials in (B.1)-(B.3), and those of (60), in (B.5) one arrives at the externalities identified in the text. \square

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