

# Voluntary Matching Grants Can Forestall Social Dumping

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## Abstract

The European economic integration leads to increasing mobility of factors. This paper investigates the possibility to achieve by means of voluntary matching grants both the optimal allocation of factors and the optimal level of redistribution in the presence of factor mobility. We use a fiscal competition model a la Wildasin (1991) in which states differ in their technologies and preferences for redistribution. We derive the optimal differentiation of matching rates across states according to the asymmetries in the technology and in the redistribution motive. Assuming that all this information is known to the states but is not known by the regulator, we derive the willingness of each state to match the contribution of other states and we decompose the aggregate willingness to pay as the sum of two terms. The first term is related to redistribution, it is positive only if matching the contribution of one state brings overall redistribution closer to its optimal level. The second term is related to production, it is positive if the matching to one state leads to a more efficient allocation of factors. Willingness to pay for matching rates converges to zero when both the optimal level of redistribution and the optimal allocation of factors are achieved. We then describe the adjustment process for the matching rates that will lead agents, informed about the technology and tastes of the other agents, to the efficient outcome and guarantee that everyone will gain. To provide additional insights on this adjustment process we develop a specific example.

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# 1 Introduction

The problem we address in this paper is the income protection of low-skilled workers in a market that is increasingly integrated. In Europe, wage subsidies have been advocated and partly implemented in some countries (France, Belgium, the Netherlands) in the form of reduced rates of employers' contributions to social security for low wages. The additional employment due to the wage subsidies in France is estimated by Crépon and Deplatz (2002) at 470.000 persons; that is about 3% of total employment in the private sector.

With the recent enlargement of the European Union, we need to address the income protection of the low-skilled in a context where there is no legal barriers to migration so that the forces of fiscal competition are at work. Following Hans-Werner Sinn (1990): "Any country that tries to establish an insurance state would be driven to bankruptcy because it would face emigration of the lucky who are suppose to give and immigration of the unlucky who are supposed to receive."

This prediction of a "race to the bottom" is too extreme; it rests on limited theoretical and empirical support. This is probably due to the presence of significant costs and barriers to migration (Welfare shopping has been discouraged in Europe by limiting portability between member states and requiring, for eligibility, previous employment in the country). However we believe that underprovision of income protection in an integrated labour market is an issue that cannot be ignored in the EU. Even if it has not been a pressing issue to date, fiscal competition for capital and labour factors is already there. And with the enlargement, this issue will become more pressing as extensively discussed in Wildasin (2004).

The objective of this paper is to propose a meaningful role for the EU in the provision of income protection of the low-skilled in the context of market integration. Our proposal is EU co-financing of national wage subsidies through a system of matching grants, with special attention to implementation.<sup>1</sup>The key questions are: could a programme of matching grants, possibly differentiated at differentiated rates, be adopted unanimously? Could it be

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<sup>1</sup>This proposal was first forcefully made by Jacques H. Drèze (2002) at the Tinbergen Lecture in Tilburg, The Netherlands.

so defined that all member states gain regardless of their differences? Could it be implemented voluntarily by member states instead of relying on the solution of a perfectly informed and powerful central planner to implement them as in the existing literature (see Wildasin , 1991).

The motivation is enhanced efficiency rather than redistribution across member states. The existing literature does not quite answer these questions, because it only provides an existence result for efficient policies, assuming (implicitly) that the net gains could be redistributed in a lump sum fashion so that everyone benefits, and that there exist a regulator with all the relevant information to implement the efficient solution. The more interesting question is whether the efficient policy could emerge from a negotiation process which simultaneously guarantees that an efficient outcome is reached and that every member state gains.

To clarify the issues, we start with a simple model proposed by Wildasin (1991) in his paper "Income Redistribution in a Common Labour Market". This model does not predict a race to the bottom but only underprovision of income support to the low-skilled. Also, Wildasin (1991) shows that when labor is mobile and each district seeks to redistribute income through specific wage subsidies it is possible to achieve the efficient allocation of labour and at the same time the optimal level of redistribution by means of differentiated matching grants. Districts with lower preference for redistribution should get higher matching grants to reduce the differences in wage subsidies, so as to minimize the distortions in the allocation of labour.

The main problem with this analysis is that local preferences for redistribution are not observable to the federal authority; still, worse the federal authority required to operate these matching grants may not exist or have the power to impose them to local authorities. The purpose of this paper is to investigate the possibility of *voluntary* matching grants among districts based on reciprocal matching; or more precisely, we investigate whether there exists some adjustment process based on each state's decisions that can bring about the optimal matching grants (i.e., those achieving both an efficient redistribution and allocation of labour).

It should be emphasized that our solution requires the participants to be informed about the technology and tastes of the other participants. This is because voluntary matching of the other participants' contributions requires to know how they would respond to such matching. This is, of course, more restrictive than one would like. However it probably more likely that participants would have this information rather than a central regulator as required in Wildasin (1991).

We use the same model as Wildasin (1991) except that districts take

into account the impact of their decision on the matching grants budget (see-through assumption). In Wildasin, the presumption is that there are enough districts for each to ignore the effects of its policy on the matching grants budget: each district takes its contribution to the matching rate policy as given when choosing its own policy. We develop our analysis in a general set up of heterogenous districts that differ both with respect to their preferences and to their technologies.<sup>2</sup>

The paper is organized as follows. Section 2 presents the framework. Pareto optimal allocations for this economy are characterized in Section 3. Section 4 studies states' willingness to match contributions of other states. Section 5 uses these findings to investigate a progressive adjustment process of matching rates so as to converge to the Pareto optimal allocation, with the property that every state is made better-off along the way. A quadratic example is explored in Section 6 to illustrate and gain further insights into this process. Finally Section 7 concludes.

## 2 The framework

A federation is composed of  $k \geq 2$  districts indexed by  $i$ . In each district there is one representative rich resident who is immobile; there are also  $l_i$  poor/workers that are mobile. Let  $L$  denote the the total number of poor/workers in the economy. Thus

$$\sum_i l_i = L. \tag{1}$$

Each district produces a private consumption good with a specific ricardian technology  $f_i(l_i)$ , which is increasing and concave ( $f'_i(l_i) > 0$  and  $f''_i(l_i) < 0$ ). Workers are paid their marginal product: wage in district  $i$  is  $w_i(l_i) = f'_i(l_i)$  which is decreasing with the number of workers in that district: ( $w'_i(l_i) = f''_i(l_i) < 0$ ).

The per capita transfer (or specific wage subsidy) that accrues to the poor (workers) in district  $i$  is denoted  $z_i$ . The total income of a poor in district  $i$  is thus  $w_i(l_i) + z_i$ .<sup>3</sup> Since poor can migrate costlessly from one district

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<sup>2</sup>The problem we address is related but more general than the voluntary matching models of Guttman (1978) or Varian (1994) who deal with simple pure public good models.

<sup>3</sup>As Alex Plekhanov pointed out, it is implicitly assumed that the firms do not pay the wage subsidy, for otherwise they would equal marginal product of labor to the wage plus the subsidy,  $f'_i(l_i) = w_i + z_i$ . However, this alternative modelling is not interesting for

to another, necessarily for any vector of transfers  $\mathbf{z} = (z_1, \dots, z_i, \dots, z_k)$ :

$$w(l_i) + z_i = w(l_j) + z_j \equiv c(\mathbf{z}) \quad \forall j, i. \quad (2)$$

which generates an allocation of labor  $\mathbf{l}(\mathbf{z}) = (l_1, \dots, l_i, \dots, l_k)$  across districts and a uniform net income of the poor  $c = c(\mathbf{z})$ . The labour demand function in district  $i$  is  $l_i(w_i) = l_i(c - z_i)$  with  $l'_i(c - z_i) = f'_i(l_i)^{-1} < 0$ . From Wildasin (1991) :

$$\frac{dc}{dz_i} = \sigma_i \in (0, 1) \quad (3)$$

where  $\sigma_i = \frac{l'_i}{\sum_j l'_j} \in (0, 1)$ ; and the general-equilibrium effect of a change in the transfer level  $z_i$  on the allocation of labour across districts is

$$\frac{dl_i}{dz_i} = -(1 - \sigma_i)l'_i > 0 \quad (4)$$

$$\frac{dl_j}{dz_i} = \sigma_i l'_j < 0. \quad (5)$$

Each district  $i$  receives  $s_i z_i l_i$  from the federation (with  $0 \leq s_i \leq 1$ ) and contributes  $\varphi_i \sum_j s_j z_j l_j$  to balance the federal budget, (with  $0 \leq \varphi_i \leq 1$  and  $\sum_i \varphi_i = 1$ ). Both subsidy rates  $s_i$  and contribution rates  $\varphi_i$  are common knowledge. The districts represented by their (immobile) rich member "see through" the central budget and capture the return to the fixed factors of production.<sup>4</sup> Hence the net income of the rich immobile resident in state  $i$  is

$$\begin{aligned} y_i &= f_i(l_i) - f'_i(l_i)l_i - (1 - s_i)z_i l_i - \varphi_i \sum_j s_j z_j l_j \\ &= f_i(l_i) - f'_i(l_i)l_i - (1 - s_i(1 - \varphi_i))z_i l_i - \varphi_i \sum_{j \neq i} s_j z_j l_j. \end{aligned} \quad (6)$$

The total effect of subsidy in state  $i$  on the net income of its rich resident  $y_i$  is obtained by totally differentiating (6) with respect to  $z_i$ , holding  $z_j$

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our purpose because the subsidy would be perceived as labor cost reducing employment without inducing migration. Thus changes in subsidy in one state will have no external effects in other states.

<sup>4</sup>In Wildasin (1991), the districts take their contribution as given and so ignore the impact of their policy choice on their contribution through the federal budget constraint.

$\forall j \neq i$  constant and using the general equilibrium effect of the migration response defined by (3)-(5).

$$\frac{dy_i}{dz_i} = -\sigma_i l_i + s_i(1 - \varphi_i)l_i + (1 - s_i)(1 - \sigma_i)z_i l'_i - \varphi_i \sum_{j \neq i} s_j z_j \frac{dl_j}{dz_i}$$

where  $\sigma_i$  is the change in the net income of the poor  $c$  resulting from an increase in  $z_i$ , and  $-(1 - \sigma_i)l'_i$  is the change in the number of poor workers resulting from this increase in  $z_i$ . The social welfare in each district  $i$  is an increasing function of the net incomes of its rich and poor residents,  $W^i(y_i, c)$ . Note that the welfare function is expressed in terms of *per capita* incomes of the rich and the poor, so that welfare is independent of the relative number of rich and poor residents which, with free migration, is endogenous to the policy choices.<sup>5</sup> It is assumed to be quasiconcave and twice differentiable with partial derivatives  $W_1^i > 0$ ,  $W_2^i > 0$ , and  $W_{11}^i \leq 0$ ,  $W_{22}^i < 0$ . Thus,  $MRS^i = W_2^i/W_1^i \geq 0$  denotes district  $i$ 's marginal willingness to pay (in income of its rich residents) for an increase in the income of its poor residents. All this information is known to all participants but is not known by the regulator.

States choose their welfare maximizing level of subsidies taking as given the subsidy choices of other states so as to attain a Nash equilibrium in subsidies such that

$$\frac{dW^i}{dz_i} = \frac{dy_i}{dz_i} + \sigma_i MRS^i = 0 \quad \forall i$$

### 3 Pareto optimality with lump-sum taxes

In this model any Pareto optimal allocation solves

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<sup>5</sup>This assumption of exogenous social welfare is again more restrictive than what one would like. However allowing for a welfare function that depends on the relative number of each group would make the analysis of the Nash equilibrium much more complicate. Roberts (1999) provides a first step in this direction by examining the process and outcomes of majority voting over public good in a club whose preferences and policy choices relate to its membership; and in turn its policy choices determine its membership. See also Drèze and Greenberg (1980) for a cooperative game approach where players's preferences are directly related to the composition of the coalition to which they belong (i.e; hedonic coalition). They showed that efficiency requires transfers across coalitions and stability requires default penalties. It is also fair to say that there is no agreement in social choice theory about how to make social welfare evaluation with variable population. In particular using the utilitarian criterion with a variable population leads to the so-called "repugnant" solution of a infinitely large population with infinitely low per capita utility.

$$\max_{l_i, y_i, c} \Lambda = \sum_i \lambda_i W^i(y_i, c) + \mu \left[ \sum_i y_i + c \sum_i l_i - \sum_i f_i(l_i) \right] + v \left[ \sum_i l_i - L \right].$$

where  $\lambda = (\lambda_1, \dots, \lambda_i, \dots, \lambda_k)$  is an arbitrary weighting system with  $\lambda_i > 0$  and  $\sum_i \lambda_i = 1$ . The necessary first-order conditions are,

$$\frac{\partial \Lambda}{\partial l_i} = \mu(c - f'_i(l_i)) + v = 0$$

$$\frac{\partial \Lambda}{\partial y_i} = \lambda_i W_1^i + \mu = 0$$

$$\frac{\partial \Lambda}{\partial c} = \sum_i \lambda_i W_2^i + \mu \sum_i l_i = 0 = \mu \sum_i \left( \frac{W_2^i}{W_1^i} - l_i \right).$$

The condition  $f'_i(l_i) = c + \frac{v}{\mu}$  is the productive efficiency condition (equalization of the marginal productivities of labour across districts). The condition  $\sum_i \left( \frac{W_2^i}{W_1^i} - l_i \right) = 0$  may be interpreted as the Bowen-Lindahl-Samuelson condition for the efficient provision of the public good  $c$ .  $\sum_i \left( \frac{W_2^i}{W_1^i} - l_i \right) > 0$  means *underprovision* and conversely.<sup>6</sup> The pareto optimal solution is denoted  $l_i^*, y_i^*, c^*$ . Wildasin (1991) shows that without matching grants, the Nash equilibrium among districts will not be efficient due to the mobility externality. Anticipating correctly the migration flows and taking the benefit levels (wage subsidies) of other districts as given, each district acting independently settles for a level of income protection that is too low (as expected from voluntary contributions to a public good); also, wages are not equalised across districts resulting in inefficient allocation of labour. Wildasin (1991) proposes a solution involving the intervention of a central regulator who can impose Pigovian taxes in the form of matching grants. He claims that there exist levels of these matching grants inducing an efficient Nash equilibrium in spite of district differences in production possibilities and preferences for redistribution. This is an existence result leaving open the question of implementation. In particular how could such matching grants be designed

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<sup>6</sup>Note that the Bowen-Lindahl-Samuelson condition does not preclude contributions to differ from willingness-to-pay for some individuals  $\frac{W_2^i}{W_1^i} - l_i \leq 0$  provided that on average the differences cancel out.

so that all districts gain, given that the regulator may not have access to all the relevant information about technology and preferences required to implement the efficient outcome.<sup>7</sup>

Our purpose is to design a decentralized mechanism that will implement the efficient allocation if the agents involved know the relevant information about all other agents. In addition to implementing efficient outcomes, the mechanism must also be acceptable to every agent. The mechanism is based on *voluntary* matching grants across agents. In this game agents set the rate at which they will match other agents' contribution and then given these matching rates they simultaneously choose their contributions.

## 4 Voluntary Matching grants

To investigate the voluntary provision of matching grants, we start by deriving the willingness-to-pay  $\pi_{ij}$  of district  $i$  for a matching rate  $s_j$  to district  $j$ . District  $i$  understands that: (i) district  $j$  will benefit from the higher  $s_j$  on the transfers  $z_j$  it pays to its workforce and will accordingly be induced to increase its own  $z_j$ ; (ii) the other districts  $k \neq j$  (including district  $i$ ) may do the same and to different extent (under asymmetry); and (iii) district  $i$  will have to pay its share of the additional cost of the matching grants resulting from the higher  $s_j$  and the higher  $z_k$ 's all around.

$$\begin{aligned} \frac{dW^i}{ds_j} &= \frac{\partial W^i}{\partial s_j} + \sum_k \frac{\partial W^i}{\partial z_k} \frac{dz_k}{ds_j} \\ &= W_1^i \left. \frac{\partial y_i}{\partial s_j} \right|_z + \sum_k \left( W_1^i \frac{\partial y_i}{\partial z_k} + W_2^i \frac{\partial c}{\partial z_k} \right) \frac{dz_k}{ds_j} \end{aligned}$$

where  $\left. \frac{\partial y_i}{\partial s_j} \right|_z = (j_{i=j} - \varphi_i) z_j l_j$  with  $j_{i=j} = 1$  if  $i = j$  and  $j_{i=j} = 0$  otherwise. Therefore, using  $\partial c / \partial z_k = \sigma_k$

$$\pi_{ij} = \frac{1}{W_1^i} \frac{dW^i}{ds_j}$$

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<sup>7</sup>Jacob Vigdor pointed out that, since the optimal solution involves a common subsidy in all states, a natural solution would be to federalize the subsidy. However without matching grants, states with different tastes and technologies would inevitably want to deviate from this solution. Thus this solution is feasible only by removing all discretion about redistributive policies from the states, which is hard to imagine, at least in the European context.

$$= (J_{i=j} - \varphi_i)z_j l_j + \frac{W_2^i}{W_1^i} \sum_k \sigma_k \frac{dz_k}{ds_j} + \sum_k \frac{\partial y_i}{\partial z_k} \frac{dz_k}{ds_j}.$$

This expression denotes the willingness-to-pay of district  $i$  for  $s_j$  taking into account the impact of  $s_j$  on district  $i$ 's contribution  $\varphi_i \sum_k s_k z_k l_k$ . Adding up over all districts the aggregate willingness-to-pay for  $s_j$  gives

$$\begin{aligned} \pi_j &= \sum_i \pi_{ij} \\ &= \left( \sum_i J_{i=j} - \sum_i \varphi_i \right) z_j l_j + \sum_i \frac{W_2^i}{W_1^i} \sum_k \sigma_k \frac{dz_k}{ds_j} + \sum_i \sum_k \frac{\partial y_i}{\partial z_k} \frac{dz_k}{ds_j} \\ &= \sum_i \frac{W_2^i}{W_1^i} \sum_k \sigma_k \frac{dz_k}{ds_j} + \underbrace{\sum_i \frac{\partial y_i}{\partial z_i} \frac{dz_i}{ds_j}}_{\text{own effect}} + \underbrace{\sum_i \sum_{k \neq i} \frac{\partial y_i}{\partial z_k} \frac{dz_k}{ds_j}}_{\text{cross effect}} \end{aligned} \quad (7)$$

where the third equality follows from the fact that  $\sum_i J_{i=j} = \sum_i \varphi_i = 1$ .

- Cross effect: for  $i \neq k$

$$\begin{aligned} M_{ik} &\equiv \frac{\partial y_i}{\partial z_k} = \frac{\partial y^i}{\partial z_k} \Big|_l + \frac{\partial y^i}{\partial l_i} \Big|_z \frac{dl_i}{dz_k} + \sum_{h \neq i} \frac{\partial y^i}{\partial l_h} \Big|_z \frac{dl_h}{dz_k} \\ &= -\varphi_i s_k l_k + \left[ -f_i'' l_i - (1 - s_i(1 - \varphi_i))z_i \right] \sigma_k l_i' \\ &\quad + \sum_{h \neq i; h \neq k} (-\varphi_i s_h z_h) \sigma_k l_h' + \varphi_i s_k z_k (1 - \sigma_k) l_k' \\ &= -\varphi_i s_k l_k - \sigma_k l_i - (1 - s_i)z_i l_i' \sigma_k - \varphi_i \sigma_k \sum_h s_h z_h l_h' + \varphi_i s_k z_k l_k' \end{aligned}$$

where the second equality follows from (4)-(5) and we have used the fact that  $l_i'(c - z_i) = f_i''(l_i)^{-1}$  in the last equation.

- Own effect: for  $i = k$ , straightforward calculation leads to

$$\frac{\partial y_k}{\partial z_k} = M_{kk} + s_k l_k + (1 - s_k)z_k l_k'.$$

Therefore using this expression and the expression for  $M_{ik}$  we have the aggregate effect,

$$\begin{aligned}
\sum_i \sum_k \frac{\partial y_i}{\partial z_k} \frac{dz_k}{ds_j} &= \sum_i \sum_k M_{ik} \frac{dz_k}{ds_j} + \sum_k \left( s_k l_k + (1 - s_k) z_k l'_k \right) \frac{dz_k}{ds_j} \\
&= - \sum_k s_k (l_k - z_k l'_k) \frac{dz_k}{ds_j} - \sum_i \left( l_i + (1 - s_i) z_i l'_i \right) \sum_k \sigma_k \frac{dz_k}{ds_j} \\
&\quad - \sum_h s_h z_h l'_h \sum_k \sigma_k \frac{dz_k}{ds_j} + \sum_k \left( s_k l_k + (1 - s_k) z_k l'_k \right) \frac{dz_k}{ds_j} \\
&= - \sum_i l_i \sum_k \sigma_k \frac{dz_k}{ds_j} + \sum_k z_k l'_k \left( \frac{dz_k}{ds_j} - \sum_h \sigma_h \frac{dz_h}{ds_j} \right). \quad (8)
\end{aligned}$$

Substituting (8) into (7) yields

$$\pi_j = \sum_i \left( \frac{W_2^i}{W_1^i} - l_i \right) \sum_k \sigma_k \frac{dz_k}{ds_j} + \sum_k z_k l'_k \left[ \frac{dz_k}{ds_j} - \sum_h \sigma_h \frac{dz_h}{ds_j} \right]. \quad (9)$$

The second term in (9) is the covariance across districts between  $z_k l'_k$  and  $dz_k/ds_j$ . Using  $\sigma_k = l'_k / \sum_h l'_h$ , this covariance can be written

$$\begin{aligned}
cov(zl', \frac{dz}{ds_j}) &\equiv \sum_k z_k l'_k \left[ \frac{dz_k}{ds_j} - \sum_h \sigma_h \frac{dz_h}{ds_j} \right] \\
&= \sum_k z_k \left[ \sigma_k \frac{dz_k}{ds_j} - \sigma_k \sum_h \sigma_h \frac{dz_h}{ds_j} \right] \sum_h l'_h \\
&= \sum_k z_k [\Delta_{kj}] \sum_h l'_h,
\end{aligned}$$

where  $\sum_k \Delta_{kj} = 0$  for all  $j$ . Letting  $\bar{z} = \sum_h \sigma_h z_h$  and rearranging we have

$$\begin{aligned}
cov(zl', \frac{dz}{ds_j}) &= \sum_h l'_h \sum_k (z_k - \bar{z}) \sigma_k \left[ \frac{dz_k}{ds_j} - \sum_h \sigma_h \frac{dz_h}{ds_j} \right] \\
&= \sum_h l'_h \sum_k (z_k - \bar{z}) \sigma_k \frac{dz_k}{ds_j}, \quad (10)
\end{aligned}$$

where the second equality follows from  $\sum_k (z_k - \bar{z})\sigma_k = 0$ . Substituting (10) into (9) and using again  $\sigma_k = l'_k / \sum_h l'_h$ ,

$$\pi_j = \underbrace{\sum_i \left( \frac{W_2^i}{W_1^i} - l_i \right) \sum_k \sigma_k \frac{dz_k}{ds_j}}_{\text{public good efficiency}} + \underbrace{(\bar{z} - z_j) \left| l'_j \right| \frac{dz_j}{ds_j} + \sum_{k \neq j} (\bar{z} - z_k) \left| l'_k \right| \frac{dz_k}{ds_j}}_{\text{production efficiency}}. \quad (11)$$

Therefore, the total willingness to pay (net of the cost) for the matching rate  $s_j$  is guided separately by the two efficiency considerations. The public good efficiency term is positive if a higher matching rate to district  $j$  can bring public good provision closer to its optimal level. Indeed since  $\sum_k \sigma_k \frac{dz_k}{ds_j} = dc/ds_j$ , this term is positive in the case of underprovision when  $dc/ds_j > 0$  and in the case of overprovision when  $dc/ds_j < 0$ . The productive efficiency term is positive if subsidizing more district  $j$  can induce a more efficient allocation of labour. The first component is the own-productivity effect of  $s_j$  and the second one is the cross-productivity effect of  $s_j$ . The own productivity effect is positive if  $(\bar{z} - z_j) \frac{dz_j}{ds_j} > 0$  so that a higher  $s_j$  induces district  $j$  to set  $z_j$  closer to the mean, implying less distortion in the allocation of labour. In addition, a higher  $s_j$  also induces a change in the choice of  $z_k$  by all other districts  $k \neq j$  with an overall reduction of the distortion in the allocation of labour if on average it reduces the spread of  $z_k$  so that  $\sum_{k \neq j} (\bar{z} - z_k) \left| l'_k \right| \frac{dz_k}{ds_j} > 0$ . To sum up,

**Proposition 1:** (a) *If productive efficiency is achieved (i.e.,  $z_k = \bar{z} \forall k$ ) and  $\sum_k \sigma_k \frac{dz_k}{ds_j} > 0$ , then the aggregate willingness-to-pay for  $s_j$  is positive in case of underprovision of the public good  $c$  and conversely. (b) *When public good efficiency is achieved, then the aggregate willingness-to-pay for  $s_j$  is positive if it brings about a more efficient allocation of labour (i.e., reallocating labour from over-employment district  $k$  where  $z_k > \bar{z}$  and  $w_k < \bar{w}$  to under-employment district  $h$  where  $z_h < \bar{z}$  and  $w_h > \bar{w}$ ).**

This proposition suggests the possibility of reaching the optimal provision of the public good and the efficient allocation of labour through some adjustment process based upon voluntary contributions.<sup>8</sup> In particular, the following MDP procedure where matching rates are treated as different public goods can achieve this outcome.<sup>9</sup>

<sup>8</sup>We emphasize that this proposition holds for any contribution rates  $\varphi_k$ 's and matching rates  $s_j$ 's.

<sup>9</sup>This procedure has been proposed independently by Malinvaud (1972) and Drèze and

## 5 Adjustment process

### 5.1 The symmetric case

The question: Could the efficient policy emerge from a negotiation process which simultaneously guarantees that an efficient outcome is reached and that every member state gains?

With identical countries it is natural to consider uniform matching grants (i.e, the Union reimburses the same fraction  $s \in [0, 1]$  of the wage subsidies issues by each country).

Suppose the initial level is  $s_o$  (possibly zero).

Suppose equal cost sharing of the matching grants and ask how much each country is prepared to contribute towards raising  $s_o$ .

Each country  $i$  understands that:

- (i) it will benefit from the higher  $s$  on the subsidies  $z_i$  it pays to its workforce;
- (ii) it will accordingly be induced to increase its own subsidies  $z_i$ ;
- (iii) the other countries will do the same and to the same extent (under symmetry);
- (iv) it will have to pay its share of the additional cost of the matching grants resulting from the higher  $s$  and the higher  $z$ 's all around.

From equation (11), setting  $z_i = z$  for all  $i$  at the symmetric equilibrium, it follows immediately that

**Proposition 2.** *Under symmetry, the sum of the voluntary contributions exceeds the corresponding cost if and only if  $s_o$  is less than the efficient level.*

An efficient outcome would result from some adjustment process in voluntary contributions that could be adopted unanimously: for instance, raise  $s$  if the sum of vountary contributions exceeds the cost and conversely if the sum of vountary contributions is less than the cost; and repeat the process until an efficient outcome is reached.

Due to symmetry, there is no budgetary appropriation: each country gets back exactly what it contributes. However the adjustment process is needed to reach the efficient outcome.

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De la Vallee Poussin (1971).

## 5.2 The asymmetric case

Looking at the structure of matching rates associated with efficient policies, we obtain a decomposition in two terms (see Appendix). The first term, *common* to all states is

$$s_i^1 = \frac{\frac{z}{w} |\varepsilon|}{\frac{z}{w} |\varepsilon| + 1} \in (0, 1)$$

where  $\frac{z}{w}$  is the share of the subsidy in the wage cost, reflecting the intensity of the preference for redistribution to the low-skilled; and  $|\varepsilon| > 0$  is the absolute value of the wage elasticity of the demand for labour, reflecting the sensitivity of the labour allocation across states to wage differentials (i.e. the mobility of labour). The matching rate is then increasing in these two variables.

This second term is a correction term *specific* to each state  $i$

$$s_i^2 = \frac{-\left(\frac{W_2^i}{W_1^i} - l_i\right)}{(L - l_i)\left(1 + \frac{z}{w} |\varepsilon|\right)}$$

where  $\frac{W_2^i}{W_1^i}$  is the marginal willingness of state  $i$  to redistribute to low-skilled, and  $l_i$  is the employment level in state  $i$ . This correction term suggests that matching rates should be differentiated according to the district preferences and technology: a higher matching rate ( $s_i^2 > 0$ ) for districts less willing to redistribute relative to their employment rate ( $\frac{W_2^i}{W_1^i} - l_i < 0$ ) and a lower matching rate ( $s_i^2 < 0$ ) for those more willing to redistribute relative to their employment rate ( $\frac{W_2^i}{W_1^i} - l_i > 0$ ). This will induce the same transfer level across districts and thus the efficient allocation of labour.

Therefore to achieve productive efficiency under asymmetry, matching rates  $s_j$  must be country-specific to induce countries to subsidize wages to the same extent. Would member states accept the principle of differentiated rates?

With differentiated rates, there are  $k$  decision variables  $s_j$  (one for each state), to which must be added  $k$  possibly different cost shares  $\varphi_j$  in the funding of the programme. So there are altogether  $2k$  decision variables to be selected so as to satisfy three conditions:

(i) productive efficiency: calling for identical wages and transfers across countries;

(ii) efficient level of public good provision (income support) as required by the Bowen-Lindahl-Samuelson condition;

(iii) individual rationality: such that every member state benefits from the programme.

In principle there are enough decision variables to satisfy the three conditions simultaneously through some adjustment process based on voluntary contributions. There is a natural adjustment process for the matching rates and compensation payments that will lead agents to the efficient outcome.

Suppose at each point in time districts announce their marginal willingness to pay for matching rates and the procedure that begins at time  $t = 0$  (with  $s_j(0) = 0 \forall j$ ) revises the matching rates and private consumption of each district according to the following system of differential equations:

$$\begin{cases} \dot{s}_j = \pi_j = \sum_i \pi_{ij} & \text{for all } j \\ \dot{y}_i = -\sum_j \pi_{ij} \dot{s}_j + \delta_i \left[ \sum_j \pi_j \dot{s}_j \right] & \text{for all } i. \end{cases}$$

That is, at each point in time, the matching rate to each district  $j$  is increased by an amount equal to the aggregate willingness to pay for this matching rate and each district  $i$  pays for this change in matching rates an amount equal to its own willingness to pay and receives a share  $\delta_i > 0$  (with  $\sum_i \delta_i = 1$ ) of the total surplus resulting from the adjustment in matching rates,  $\sum_j \pi_j \dot{s}_j = \sum_j \pi_j^2 > 0$ . This procedure has several desirable properties under correct revelation. First under quasi-linear preferences (no income effect), it is making every state better off at each point in time.<sup>10</sup> Indeed letting  $V^i(y_i, \mathbf{s})$  denote the (indirect) utility function of each district  $i$  as a function of its net income and of the matching rates,

$$\begin{aligned} \frac{dV^i}{dt}(y^i, \mathbf{s}) &= \left( y_i + \sum_j \pi_{ij} \dot{s}_j \right) \frac{\partial V^i}{\partial y^i} \\ &= \delta_i \left( \sum_j \pi_j \dot{s}_j \right) \frac{\partial V^i}{\partial y^i} \\ &= \delta_i \left( \sum_j \pi_j^2 \right) \frac{\partial V^i}{\partial y^i} > 0 \quad \text{for } \delta_i > 0. \end{aligned}$$

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<sup>10</sup>The restriction to quasi-linear preferences is needed to prevent the (Nash) equilibrium choice of  $z_i$  to be affected by the redistribution of the surplus resulting from the adjustment process. With quasi-linearity we have  $\partial z_i / \partial y^i = 0$  which implies non-distortionary redistribution of the surplus.

Second, the stationary solution of the procedure is a Pareto optimum since then  $\dot{s}_j = \pi_j = 0$  for all  $j$ . The monotonicity of the utilities implies the convergence of the process. The convergence of the procedure to a stationary solution is proved by taking the sum of utilities as the Lyapunov function,

$$L(t) = \sum_i V^i(t)$$

which is monotonically increasing with derivative equal to zero only at a stationary point. The strict concavity of  $V^i$  implies the global convergence of the process to a unique stationary point.

This still leaves open a crucial question however. Why should the regions submit their true preferences and technologies to the centre? Might it not sometimes pay to misrepresent one's preference and technology? The answer is that it could, but we have ruled out this possibility by assuming participants to be informed about the technology and tastes of other participants; even if there is no regulator possessing such information. Relaxing this assumption is a formidable task in two respects: first participants would not know how others would respond to their matching rate which is required to calculate their willingness to pay for matching rates; second each participant can gain from misrepresenting his willingness to pay to manipulate the adjustment process to his own advantage (e.g. by claiming low tastes for redistribution to receive higher matching grants). The second issue has been addressed in the literature for a simple public good problem where agents announce their willingness to pay and the centre provides directly the public good. For instance, Drèze and De la Vallee Poussin (1971, Theorem 3) have shown that truthful revelation at each point in time is a maximin strategy (i.e., the best response to the most unfavourable strategies of the other players). Revelation in dominant strategy is more problematic (See Laffont, 1988, chapter 5). The main result here is that truthful revelation is a dominant strategy at a stationary point (see Drèze and De la Vallée Poussin, 1971). Even if participants misrepresent their preference, Roberts (1979) has shown that the MDP procedure still generate Pareto optimal outcomes. The effect of preference manipulation is simply to slow down the adjustment process. Addressing the two issues simultaneously has never been done and is a very difficult task.

[INSERT ROBERT' LIKE PROOF ABOUT HERE]

There is also an obvious difficulty in solving the system of differential equations, given the complexity of expressions. To obtain additional insights, therefore, we compare two districts with specific production and utility functions. The purpose is to illustrate how the adjustment process will

effectively guide the agents to the efficient outcome through the progressive adjustment of the matching rates. We shall also carry out some comparative statics analysis with respect to the asymmetry between districts (both in terms of preferences and technologies).

## 6 Example

Consider two districts  $i = 1, 2$  with utility function  $W^i(y_i, c) = y_i + (c - \frac{m_i}{2}c^2)$  and technology  $f_i(l_i) = l_i - \frac{b_i}{2}l_i^2$  (with  $(b_i, m_i) \in (0, 2) \times (0, 2)$ ). The total allocation of labour between the two districts is  $l_1 + l_2 = L = 1$ . Given matching rates  $s_i \in [0, 1]$  and tax shares  $\varphi_i \in (0, 1)$  (with  $\varphi_1 + \varphi_2 = 1$ ) the net income of the rich in district  $i$  is

$$\begin{aligned} y_i &= f(l_i) - l_i f'_i(l_i) - (1 - (1 - \varphi_i)s_i)l_i z_i - \varphi_i s_j l_j z_j \\ &= \frac{b_i}{2}l_i^2 - (1 - (1 - \varphi_i)s_i)l_i z_i - \varphi_i s_j l_j z_j. \end{aligned}$$

From the (interior) equilibrium migration  $f'_i(l_i) + z_i = f'_j(l_j) + z_j = c$ , we have

$$l_i = \frac{b_j}{b_i + b_j} + \frac{z_i - z_j}{b_i + b_j}.$$

Therefore

$$\frac{\partial l_i}{\partial z_i} = -\frac{\partial l_i}{\partial z_j} = \frac{1}{b_i + b_j} > 0$$

and

$$\frac{\partial c}{\partial z_i} = 1 + \frac{\partial w_i}{\partial l_i} \frac{\partial l_i}{\partial z_i} = \frac{b_j}{b_i + b_j}.$$

The Pareto optimal solution is characterized by

$$\begin{cases} f'_i(l_i) = f'_j(l_j) = f'_2(1 - l_i) \\ W_2^i + W_2^j = l_i + l_j = 1 \end{cases}$$

from which it follows that

$$\begin{aligned} \widehat{l}_i &= \frac{b_j}{b_i + b_j} \\ \widehat{z} &= \frac{1 - (m_i + m_j)}{m_i + m_j} + \frac{b_i b_j}{b_i + b_j} \\ \widehat{c} &= \frac{1}{m_i + m_j}. \end{aligned}$$

In a Nash equilibrium each district  $i$  chooses  $z_i$  taking  $z_j$  as given so as to maximize,

$$W^i(y_i, c) = \frac{b_i l_i^2}{2} - [1 - (1 - \varphi_i) s_i] l_i z_i - \varphi_i s_j l_j z_j + (c - \frac{m_i}{2} c^2).$$

The necessary first-order condition for each district  $i = 1, 2$  is :

$$\frac{\partial W^i}{\partial z_i} = \frac{b_i l_i}{b_i + b_j} - (1 - (1 - \varphi_i) s_i) (l_i + \frac{z_i}{b_i + b_j}) + \frac{\varphi_i s_j z_j}{b_i + b_j} + (1 - m_i c) \frac{b_j}{b_i + b_j} = 0.$$

The second-order condition<sup>11</sup> is satisfied for  $\varphi_i = 1/2$ . The optimal differentiation of matching rates is obtained by setting  $z_i = \hat{z}$ ,  $l_i = \hat{l}_i$  and  $c = \hat{c}$ , and subtracting the first-order conditions of both districts; this yields

$$\frac{b_j s_i - b_i s_j}{2} = \frac{m_i b_j - m_j b_i}{m_i + m_j}.$$

Therefore, with identical technologies ( $b_i = b_j$ ), the matching rate should be higher for the district less willing to redistribute:  $m_i > m_j \iff s_i > s_j$ , and with identical preferences ( $m_i = m_j$ ), the matching rate should be higher for the more productive district:  $b_i < b_j \iff s_i > s_j$ . The first-order conditions describe a system of two equations of the form

$$\begin{cases} A_i z_i + B_i z_j + C_i = 0 \\ A_j z_j + B_j z_i + C_j = 0 \end{cases}$$

for which the solution is

$$\begin{aligned} z_i &= \frac{B_i C_j - A_j C_i}{A_i A_j - B_i B_j} \\ z_j &= \frac{B_j C_i - A_i C_j}{A_i A_j - B_i B_j}. \end{aligned}$$

where<sup>12</sup>

$$A_i = \frac{b_i}{b_i + b_j} - 2(1 - (1 - \varphi_i) s_i) - m_i b_j \left(1 - \frac{b_i}{b_i + b_j}\right)$$

<sup>11</sup>For  $\varphi_i = 1/2$ , we have that for all  $s_i \in [0, 1]$

$$\frac{\partial^2 W^i}{\partial z_i^2} = \left(\frac{1}{b_i + b_j}\right) \left(\frac{b_i}{b_i + b_j} - 2\left(1 - \frac{s_i}{2}\right)\right) - m_i \left(\frac{b_j}{b_i + b_j}\right)^2 < 0.$$

<sup>12</sup>We make the generically innocuous assumption that the matrix of coefficients of the system of equations is non-singular (i.e,  $A_i A_j - B_i B_j \neq 0$ ).

$$\begin{aligned}
B_i &= -\frac{b_i}{b_i + b_j} + [1 - (1 - \varphi_i) s_i] + \varphi_i s_j - \frac{m_i b_i b_j}{b_i + b_j} \\
C_i &= \frac{b_i b_j}{b_i + b_j} - (1 - (1 - \varphi_i) s_i) b_j + b_j - m_i b_j \left(1 - \frac{b_i b_j}{b_i + b_j}\right).
\end{aligned}$$

We would like now to use this example to see how the adjustment process will effectively guide the Nash equilibrium to the optimal solution through the progressive adjustment of the matching rates. Specifically we are interested to solve (numerically) the system of differential equations

$$\begin{cases} \dot{s}_1 = \pi_1 \\ \dot{s}_2 = \pi_2 \end{cases}$$

with

$$\begin{aligned}
\pi_j &= \sum_{i=1}^2 \left( \frac{W_2^i}{W_1^i} - l_i \right) \sum_{k=1}^2 \sigma_k \frac{dz_k}{ds_j} + \sum_{k=1}^2 (\bar{z} - z_k) \left| l'_k \right| \frac{dz_k}{ds_j} \\
&= \sum_{i=1}^2 (1 - m_i c - l_i) \sum_{\substack{k=1 \\ i \neq k}}^2 \frac{b_i}{b_i + b_k} \frac{dz_k}{ds_j} + \sum_{k=1}^2 \frac{(\bar{z} - z_k)}{b_k} \frac{dz_k}{ds_j}
\end{aligned}$$

where

$$c = w_i + z_i = (1 - b_i l_i) + z_i,$$

$$l_i = \frac{b_k}{b_i + b_k} + \frac{z_i - z_k}{b_i + b_k}, \quad k \neq i$$

$$\bar{z} = \frac{b_i}{b_i + b_k} z_k + \frac{b_k}{b_i + b_k} z_i, \quad k \neq i.$$

## 6.1 Symmetric case

In the case of identical districts, the symmetric Nash equilibrium is given by<sup>13</sup>

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<sup>13</sup>From the resource constraint, non-negativity of  $y$  in the symmetric Nash equilibrium requires  $y = f(1/2) - \frac{1}{2}c = \frac{1}{2}(1 - \frac{b}{4} - \frac{1}{2m}) > 0$  which implies that  $m > \frac{1}{2-b/2}$ . Combining this requirement with the non-negativity of  $z$  we have  $m \in (\frac{1}{2-b/2}, \frac{1}{2-b})$ .

$$z(s) = \frac{\frac{1}{2}(1+s) - (1-\frac{b}{2})m}{\frac{1-s}{b} + m} > 0 \text{ for } m < \frac{1}{2-b}.$$

It follows that  $\partial z/\partial s > 0$ ,  $\partial z/\partial b > 0$ , and  $\partial z/\partial m < 0$ . The optimal (uniform) matching rate is,

$$\begin{aligned} \hat{s} &= \frac{z|\varepsilon|}{z|\varepsilon| + w} \\ &= \frac{1 - (2-b)m}{1 - (2-b)m + bm} \in (0, 1) \text{ for } m < \frac{1}{2-b} \end{aligned}$$

which is increasing in  $b$  and decreasing in  $m$ . The following tables illustrate the effects of a change in the preferences and technology on the (symmetric) Nash equilibrium level of transfer in the absence of matching grants  $z_0$  and on the optimal level of transfer  $\hat{z}$  and matching rate  $\hat{s}$ .

**Table 1a. Effect of a change in preference ( $b = 0.5$ )**

| $m$       | 0.58     | 0.6      | 0.62     | 0.64      | 0.66      |
|-----------|----------|----------|----------|-----------|-----------|
| $\hat{z}$ | 0.112069 | 0.083333 | 0.056451 | 0.0312502 | 0.0075757 |
| $z_0$     | 0.025193 | 0.019230 | 0.013358 | 0.0075757 | 0.0018797 |
| $\hat{s}$ | 0.309524 | 0.250000 | 0.184212 | 0.1111120 | 0.0294118 |

**Table 1b. Effect of a change in technology ( $m = 1.$ )**

| $b$       | 1.1      | 1.2      | 1.3      | 1.4      | 1.5      |
|-----------|----------|----------|----------|----------|----------|
| $\hat{z}$ | 0.050000 | 0.100000 | 0.150000 | 0.200000 | 0.250000 |
| $z_0$     | 0.026190 | 0.054545 | 0.084782 | 0.116660 | 0.150000 |
| $\hat{s}$ | 0.083333 | 0.142857 | 0.187500 | 0.222222 | 0.250000 |

The following figure describes the (numerical) solution  $s(t)$  to the differential equation characterizing the adjustment procedure given the symmetry between districts (with  $b = 0.5$  and  $m = 0.6$ ). It shows clearly the convergence of the matching rate to its optimal level  $\hat{s} = 0.25$

[Insert figure 1]

## 6.2 Asymmetric case

- **Preference asymmetry** ( $m_1 = 0.58 < m_2 = 0.66$ )

Fixing  $b_1 = b_2 = 1.4$ , the Nash equilibrium without matching rates is  $z_1 = 0.02099 > z_2 = 0.00572$  while the optimal solution is  $\hat{z}_1 = \hat{z}_2 = 0.05645$ . The optimal matching rates are  $\hat{s}_1 = 0.119694 < \hat{s}_2 = 0.24872$ . In words, district 2 has a weaker redistribution concern and should receive a higher matching rate to choose the same transfer as district 1 (as required for optimality). The figure below depicts the solution  $(s_1(t), s_2(t))$  to the system of differential equations characterizing the MDP procedure for the parameter values chosen. Again, we see that matching rates are converging to their optimal values. It is worth noting that in this example, district 1 (with a stronger preference for redistribution) is actually making a net transfer to district 2 (with a weaker preference for redistribution):  $(\hat{s}_1 \hat{l}_1 - \hat{s}_2 \hat{l}_2) \hat{z} < 0$ , where  $\hat{l}_1 = \hat{l}_2$  since technology is the same.

[Insert figure 2]

- **Technology asymmetry** ( $b_1 = 1.1 < b_2 = 1.4$ )

Fixing  $m_1 = m_2 = 1$ , the Nash equilibrium without matching rates is  $z_1 = 0.03111 < z_2 = 0.09778$  (it is more costly for the more productive and high-employment district 1 to subsidize its labour). The optimal solution is  $\hat{z}_1 = \hat{z}_2 = 0.116$  which requires optimal matching rates of  $\hat{s}_1 = 0.25945$  and  $\hat{s}_2 = 0.05748$  (the high-employment district 1 should receive a higher matching rate). The solution  $(s_1(t), s_2(t))$  to the MDP procedure is given in the following figure. Notice that in this example at the optimum, the poor district 2 is making a net transfer to the rich district 1, since  $(\hat{s}_1 \hat{l}_1 - \hat{s}_2 \hat{l}_2) \hat{z} > 0$  with  $\hat{l}_1 > \hat{l}_2$ .

[Insert figure 3]

## 7 Conclusion

The recent European enlargement is the largest single expansion that the European Union has ever experienced, with ten countries and 73 million

people joining the club. It is not just the largest EU expansion, but also the most diversifying; the gaps between the living standards and economic structures of existing EU nations and those that are joining are far wider than in previous enlargements. One of the great prizes of EU membership for people from the new countries is the right to live and work in the rest of the EU. Although East European economies have been growing rapidly in the past ten years, average wages are still only 12 percent of those of Britain. Granting immediate employment and residential rights and the full access to the welfare state could prove a very powerful pull factor. Some economists have said that open-border immigration policy can be incompatible with a welfare state and that it will trigger a race to the bottom. Other pro-immigration economists believe that it will attract workers who are needed in key sectors and that it will not be a burden on the public purse. For employers mobility enables recruitment from a wider pool of people and helps to alleviate regional skills shortage. It will attract skills and boost the economy.

In this context, we have examined a fiscal competition game in which the contribution by one state to support the income of its low-skilled workers may affect other states through the induced migration. Due to this fiscal externality the Nash equilibrium is typically inefficient: there is too little income protection for the low-skilled workers due to the fear of immigration; and different states will choose different redistributive policies so that wages are not equalised across districts resulting in inefficient allocation of labour. To achieve the efficient allocation, each state must face the correct "price" for its choice. Wildasin (1991) proposes a solution involving differentiated matching grants. He claims that there exist levels of these matching grants inducing an efficient Nash equilibrium in spite of district differences in production possibilities and preferences for redistribution. The problem is how to determine the correct prices so that all states would benefit given that the regulator may not have access to all the information about technology and preferences to implement efficient outcome.

Our purpose has been to design a decentralized mechanism that will implement the efficient allocation if the agents involved know the relevant information about all other agents. In addition to implementing efficient outcomes, the mechanism should be acceptable to every agents. The mechanism is based on *voluntary* matching grants across agents. This is a mechanism where agents set the rate at which they will match other agents' contribution and then given these matching rates they simultaneously choose their contributions and collect the promised subsidies from the other agents. We have examined some adjustment process capable of leading the agents to the

efficient solution. According to this process the matching rates are progressively adjusted based on what agents are willing to pay and compensations are paid so that everyone gains. An example is developed to provide additional insights in the adjustment process.

## 8 Appendix

We must characterize the optimal matching rates when districts may differ with respect to their preference and technologies. We make the following three assumptions:

(i) *at the optimum allocation of labour* ( $l_i = l_i^*$ ) the wage elasticity of labour demand is the same across districts

$$\varepsilon_i \equiv \frac{w^*}{l_i^*} l_i^{*'} = \epsilon \quad \forall i;$$

(ii) contribution rates are based on optimal shares in total employment

$$\varphi_i = \frac{l_i^*}{L};$$

(iii) each district  $i$  ignores the migration-induced effect of its choice of  $z_i$  on its contribution  $\varphi_i \sum_j s_j z_j \frac{dl_j}{dz_i}$ .<sup>14</sup>

Using these simplifications and the productive efficiency condition, each district  $i$ 's FOC on  $z_i$  is

$$\sigma_i \left( \frac{W_2^i}{W_1^i} - l_i \right) + s_i (1 - \varphi_i) l_i + (1 - s_i) (1 - \sigma_i) z_i l_i' = 0$$

leading to

$$s_i = \frac{-(1 - \sigma_i) z_i l_i' - \sigma_i \left( \frac{W_2^i}{W_1^i} - l_i \right)}{(1 - \varphi_i) l_i - (1 - \sigma_i) z_i l_i'} := s_i^1 + s_i^2.$$

From the above assumptions,  $\sigma_i = \frac{l_i'}{\sum_j l_j'} = \frac{l_i \varepsilon}{w} / \sum_j \frac{l_j \varepsilon}{w} = \frac{l_i}{L} = \varphi_i$  and therefore,

$$s_i^1 = \frac{z |\varepsilon|}{z |\varepsilon| + w}$$

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<sup>14</sup>The same assumption is made in Wildasin (1991). Note also that this assumption is irrelevant with identical districts and matching rates because  $\sum_j s_j z_j \frac{dl_j}{dz_i} = sz \sum_j \frac{dl_j}{dz_i} = 0$ .

$$s_i^2 = \frac{-(\frac{W_2^i}{W_1^i} - l_i)}{(L - l_i)(1 + \frac{z}{w} |\varepsilon|)}.$$

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